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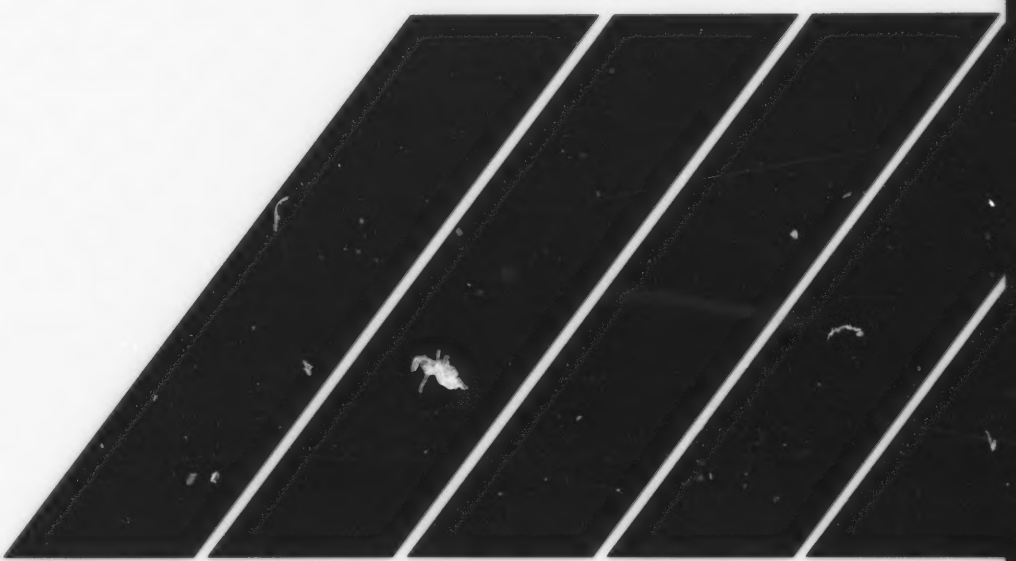
Efficiency and Alternative Energy Technology Branch

Direction de la technologie de l'efficacité énergétique et des énergies de remplacement



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DEVELOPMENT OF ALGORITHMS FOR GSHP HEAT EXCHANGER LENGTH PREDICTION AND ENERGY ANALYSIS

PREPARED FOR:

Efficiency and Alternative Energy Technology Branch
Energy, Mines and Resources Canada
Ottawa, Ont.
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FOREWARD

This project was initiated to determine the suitability of the commonly used algorithms for sizing ground heat exchangers in ground-source heat pump applications. Many ground-source heat pump applications in Canada involve both heating and cooling, i.e., the ground heat exchanger absorbs heat from the earth in winter and dumps heat to the earth in summer. The suitability of sizing algorithms was evaluated in terms of the effect of heat exchanger length on system performance and in terms of installations cost. There is a need to develop validated heat exchanger sizing and energy analysis procedures for ground-source heat pumps. Designers and contractors need more accurate methods to optimize the heat exchanger length to make the systems more competitive. This information is especially critical in commercial installations where the length of heat exchanger can be quite long.

Gathering monitored data was difficult. We were not successful in obtaining data for large commercial installations or lake loop projects. However, the data obtained enabled the project team to determine that the most commonly used sizing algorithm gives oversized ground heat exchangers and sometimes by an amount to have a significant cost penalty. The system performance does not show a significant penalty, at least in the residential applications studies in this project. Commercial applications need to be studied before a general conclusion can be reached about the effect of sizing on system performance.

The preferred sizing algorithm was then transformed into time step formulation suitable for computerized implementation. This algorithm will be incorporated into the EMR Hot 2000™ computer program. Other building simulation program developers will be informed of the study with the objective of improving the treatment of ground heat exchanger sizing.

More effort is required to validate models for commercial and institutional buildings. Further work in the development of models for lake loop and spiral heat exchangers is also recommended. The project Steering Committee recommends the implementation of the validated model into a heat exchanger sizing methodology or computer program suitable for residential contractors. A further recommendation is to have the procedure mandated by including it as part of the CSA Design and Installation Standard, CSA-C445.

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Steve Trelease, Command-Aire Corporation
John Andrews, Brookhaven National Laboratory

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EXECUTIVE SUMMARY

This report presents the development and validation of a number of ground-source heat pump heat exchanger models for residential systems. The mathematical development of the models is presented in Appendices A and B of the report.

The vertical ground heat exchanger configurations for which models were developed were single u-tube, two u-tubes, three u-tubes and four u-tubes in a square pattern. The horizontal ground heat exchanger models are single pipe in a trench, two pipes in a trench at the same depth, two pipes in a trench at different depths and four pipes in a trench on a square pattern. A model for lake-loop systems was developed and is described in Appendix B.

The models validated by comparison to real data were the single pipe in a trench, two pipes in a trench (over/under), two pipes in a trench (side-by-side) and three vertical u-tubes in parallel. Data from field tests performed by W.S. Fleming and Associates for Niagara Mohawk Corporation and Brookhaven National Laboratory were used in the comparisons. The models developed here predicted the actual heat exchanger lengths within 13 percent in the case of horizontal configurations, while the vertical case was within 1 percent. The length procedure used by industry and published by the International Ground Source Heat Pump Association was, at best, within 20 percent in the vertical case, but anywhere from 70 to 130 percent longer lengths were predicted in the horizontal cases.

The most important implication of oversizing the ground heat exchanger is the significant increase in installed cost and the degraded cost/benefit to the customer. By lowering the installed cost, the new prediction models, if widely adopted, should broaden the potential market for ground source heat pumps in Canada.

The models developed here are being implemented into EMR's Hot™ 2000 computer program. More validation effort is required to determine if the models are suitable for commercial or institutional buildings. Further work in development of models for lake-loop and spiral heat exchangers is also recommended. A consensus reached by the project Steering Committee calls for the implementation of the model developed here into a heat exchanger sizing methodology or computer program suitable for residential contractors. The next step would be to have the procedure mandated by including it as part of the CSA Design and Installation Standard, CSA-C445.

SOMMAIRE

Ce rapport présente le développement et la validation de différents modèles d'échangeurs de chaleur pour des systèmes résidentiels de pompes à chaleur à réseaux enterrés. La formulation mathématique de ces modèles est documentée dans les annexes A et B de ce rapport.

Les échangeurs de chaleur pour réseaux enterrés de tubes verticaux (puits forés), pour lesquels les modèles ont été développés, sont: un tube de forme u dans un puit foré (tube-u), deux tubes-u, trois tubes-u, et quatre tubes-u dans une géométrie rectangulaire. Les différents échangeurs de chaleur pour réseaux enterrés de tubes horizontaux sont de type: un tube dans une tranchée, deux tubes dans une tranchée à differentes profondeurs, deux tubes dans une tranchée enterrés à la même profondeur, et finalement quatre tubes dans une tranchée arrangés dans une géométrie rectangulaire. Un modèle développé pour des système fonctionnant en boucle fermée avec des réservoirs d'eau est décrit dans l'annexe B.

Les modèles utilisés durant la phase de validation avec des données réelles sont, un tube dans une tranchée, deux tubes dans une tranchée enterrés à différentes profondeurs (aussi nommé dessus/dessous), deux tubes dans une tranchée enterrés à la même profondeur (côte-à-côte), et enfin trois tubes-u installés en parallèle. Les résultats d'études expérimentés in-situ faites par W.S. Fleming and Associates pour la Niagara Mohawk Corporation et par le Brookhaven National Laboratory ont été utilisés pour la validation des modèles développés dans cette étude. Ces derniers ont prédit à treize pourcent près les longueurs actuelles des échangeurs de chaleur pour réseaux horizontaux, et à un pourcent près dans le cas du système vertical. La méthode de calcul utilisée par l'industrie et, publiée par l'association IGSHPA, International Ground Source Heat Pump Association, a prédit, quant à elle, à vingt pourcent près la longueur des tubes dans le cas du système vertical. En ce qui concerne les systèmes horizontaux, les prédictions des longueurs des tubes étaient de soixante-dix à cent trente pourcent plus importantes que les longueurs actuelles.

L'impact majeur du surdimensionnement des échangeurs de chaleur enterrés est l'augmentation significative du coût d'installation et donc d'une dégradation du rapport coût/bénéfices pour l'acheteur. En diminuant le coût d'installation, les nouveaux procédés de calcul, si ils sont utilisés à grande échelle, devraient élargir le marché potentiel des pompes à chaleur à réseaux enterrés au Canada.

Les modèles, sujets de ce rapport, sont en train d'être incorporés dans le logiciel HOT™ 2000 de EMR. Une autre phase de validation est nécessaire afin de déterminer si ces modèles sont compatibles pour des systèmes utilisés dans des édifices commerciaux ou institutionels. D'autres études sont recommandés afin de développer des modèles pour les échangeurs de chaleur de type boucle d'eau sur réservoir d'eau et de type spirale. Un accord établi par le Comité de Direction du Projet demande l'incorporation de ces modèles dans une méthode de calcul, ou dans un logiciel, déterminant la longueur des échangeurs de chaleur, appropriés pour l'utilisation par les entrepreneurs résidentiels. L'étape suivante serait d'avoir la méthode de calcul mandaté en l'incorporant au standard, CSA Design and Installation Standard, CSA-C445.

CONTENTS

	PAGE
ACKNOWLEDGEMENTS	ii
EXECUTIVE SUMMARY	iii
CONTENTS	v
LIST OF FIGURES	vi
LIST OF TABLES	vii
1. INTRODUCTION AND BACKGROUND	1
2. THE MODELS	1
3. COMPARISON BETWEEN REAL DATA AND MODEL PREDICTIONS	2
3.1 Descriptions of the Installations	2
3.2 Results of Comparisons - Length Prediction	3
3.3 Results of Comparisons- Energy Analysis	4
4. CONCLUSIONS	5
5. RECOMMENDATIONS	6
6 REFERENCES	7
APPENDIX A Description of the Mathematical Models for Ground Heat Exchanger Length Prediction	
APPENDIX B Description of the Mathematical Models for Lake-Loop Heat Exchanger Length Prediction	

LIST OF FIGURES

		page
Figure 1	Model Validation #1 - Earth Loop Configuration : Single Pipe Horizontal	9
Figure 2	Model Validation #1 - Single Pipe Horizontal : Measured vs. Predicted Entering Water Temperature	10
Figure 3	Model Validation #2 - Earth Loop Configuration : Two Pipe Over/Under Horizontal	12
Figure 4	Model Validation #2 - Two Pipes Horizontal : Measured vs. Predicted Entering Water Temperature	13
Figure 5	Model Validation #3 - Earth Loop Configuration : Two Pipe Side-By-Side Horizontal	15
Figure 6	Model Validation #3 - Two Pipes Horizontal : Measured vs. Predicted Entering Water Temperature	16
Figure 7	Model Validation #4 - Earth Loop Configuration : 3 U-Tubes Parallel Vertical	18
Figure 8	Model Validation #4 - 3 U-Tubes Vertical : Measured vs. Predicted Entering Water Temperature	19

LIST OF TABLES

		page
Table 1	Model Validation #1 - Parameters Values for Comparison Run : Single Pipe Horizontal	8
Table 2	Model Validation #2 - Parameters Values for Comparison Run : Two Pipe Over/Under Horizontal	11
Table 3	Model Validation #3 - Parameters Values for Comparison Run : Two Pipe Side-By-Side Horizontal	14
Table 4	Model Validation #4 - Parameters Values for Comparison Run : 3 U-Tubes Parallel Vertical	17
Table 5	Model Validation - Comparison Runs : Total Pipe Length Predicted vs. Actual	20
Table 6	Model Validation #2 - Energy Analysis Comparison : Two Pipe Over/Under Horizontal - Prediction with Actual Length	21
Table 7	Model Validation #3 - Energy Analysis Comparison : Two Pipe Side-By-Side Horizontal - Prediction with Actual Length	22
Table 8	Model Validation #4 - Energy Analysis Comparison : 3 U-Tubes Parallel Vertical - Prediction with Actual Length	23
Table 9	Model Validation #2 - Energy Analysis Comparison : Two Pipe Over/Under Horizontal - Prediction with Actual Minimum EWT	24
Table 10	Model Validation #3 - Energy Analysis Comparison : Two Pipe Side-By-Side Horizontal - Prediction with Actual Minimum EWT	25

Table 11 Model Validation #4 - Energy Analysis Comparison : 3 U-Tubes Parallel Vertical - Prediction with Actual Minimum EWT 26

Table 12 Model Validation #3 - Energy Analysis Comparison : Two Pipe Side-By-Side Horizontal - Prediction with Increased Load (and Actual Length) 27

1. INTRODUCTION AND BACKGROUND

There has been a considerable need, for sometime, to develop updated, validated, heat exchanger sizing and energy analysis procedures for ground-source heat pumps (GSHPs). Designers and contractors need more accurate methods to optimize the amount of ground-coil installed and thereby make the systems more competitive.

Considerable evidence exists to suggest that current procedures used by industry oversize both vertical and horizontal systems. Caneta Research [1] published a paper describing their efforts on behalf of Ontario Hydro to identify suitable methods for predicting performance of GSHP systems. The appendix of the paper described in detail two methods used to model GSHPs. The method published by the National Water Well Association (NWWA) in 1986 [2] was compared to that contained in the ASHRAE Design Data Manual [3]. The NWWA method compared favourably with field observed data reported by Hughes et al [4] while the ASHRAE Method [3] did not.

The objective of this project was to develop updated, validated, sizing procedures for GSHP systems. These models were to be incorporated into EMR's Hot 2000™ computer program and the results were to be published to permit other software developers to incorporate them into their programs.

This project would develop a number of models of both vertical and horizontal heat exchanger configurations, with latent heat approximations included as well as a model for lake-loop heat exchanger sizing. The models were to be tested against real field performance data, where available. The models were to be documented to permit Fortran coding by the Hot 2000™ software developers.

2. THE MODELS

The closed-loop models documented by the project and described in detail in Appendix A included the following ground heat exchanger configurations.

Vertical

- single u-tube
- 2 u-tubes
- 3 u-tubes
- 4 u-tubes (in square)

Horizontal

- single pipe in trench
- two pipes in trench (same depth)
- two pipes in trench (different depth)
- four pipes in trench (in square)

The Kelvin Line Source heat transfer theory is used to predict time-dependent heat transfer from or to a line source (heat exchanger) in the earth. Resistance to heat transfer through the soil and the temperature distribution around the heat exchanger can be predicted. The effects of cyclic on-off operation are accounted for, as are the thermal interference effects of other nearby heat exchanger pipes. The effect of seasonal far-field earth temperature variation with depth is also accounted for. A soil moisture freezing approximation is made, in horizontal models, by assuming that a frozen earth ring or radius surrounds the pipe, with a thermal conductivity equivalent to ice and a surface temperature of $0^{\circ}C$.

The effects of local climate and soil type can also be accounted for. Soil thermal properties are required to be specified, as are the local soil characteristics such as the undisturbed mean earth temperature, the annual earth surface temperature amplitude and the earth surface phase constant.

The lake-loop model assumes a lake which is shallow (< 10 m) and unstratified, with very slow temperature changes. The model accounts for the effects of solar radiation, evaporative heat losses, convective heat losses or gains from or to the lake, ground heat flows at the lake boundaries and the heat removal or addition due to the lake-loop heat pump heat exchanger. The model and input data requirements are described in detail in Appendix B.

3. COMPARISON BETWEEN REAL DATA AND MODEL PREDICTIONS

This section presents the results of a comparison between real data from field monitored ground-source heat pump installations and the predictions of the closed-loop ground heat exchanger models for 4 separate cases. No real data could be identified for validation of the lake-loop model during this project.

The sources of the data used here are published reports prepared by Brookhaven National Laboratory [5] and by W.S. Fleming and Associates [6] for Niagara Mohawk Corporation.

3.1 Descriptions of the Installations

Tables 1, 2, 3 and 4 present data that was used as input for the model runs. Figures 1, 3, 5 and 7 provide a sketch of the earth loop configurations as modelled in each of the cases examined.

The four configurations used in the comparison were:

- single pipe-per-trench horizontal (serpentine);
- two pipe-per-trench horizontal (over/under),
- two pipe-per-trench horizontal (side-by-side);
- 3 vertical U-tubes in parallel.

The single pipe-per-trench horizontal system data was taken from the P.D. Metz work at Brookhaven National Laboratory (BNL) [5]. The remaining three earth-loop configurations were from the W.S. Fleming and Associates field monitoring project [6].

Heat Pump performance data was obtained from the manufacturer or from performance data provided by F. Lenarduzzi of Ontario Hydro. Soil property data were taken from the ASHRAE Design and Data manual based on soil descriptions in the report or obtained through contact with the original investigators. Where available, the soil far-field or undisturbed soil temperature measurements on-site were used to adjust the published normal soil temperature characteristics such as the mean soil temperature, amplitude of surface soil temperature and the phase constant.

The actual monitored heat extraction/rejection rates or run-time ratios (i.e. load on the ground heat exchanger) were used to drive the model predictions where the heating and cooling loads met by the heat pumps were not measured.

3.2 Results of Comparisons - Length Prediction

Generally, residential ground-source heat pump heat exchangers in Canada are sized so that the liquid temperature returning to the heat pump does not drop below a particular value. This is normally done to ensure that the operating COP of the heat pump does not drop below a particular value.

Comparisons of the measured entering water temperature (EWT) and that predicted by the current models under development here based on the NWWA method and the conventional methods published by ASHRAE [3] or IGSHPA [7] are presented in Figures 2, 4, 6 and 8. Also shown in Figures 2, 4 and 6, for the three horizontal earth-loop configurations, are predictions with and without the effect of soil freezing. The soil moisture freezing approximation is that reported by Hart and Couvillion in their work for the National Water Well Association published in 1986. It is evident from the three plots that the predictions with the soil freezing approximation give better agreement, particularly

in the periods with the lowest EWTs. The predictions using the models developed here are generally within 1 °C of the measured EWTs.

Table 5 presents "actual" and model "predicted" lengths at the measured minimum EWT for each of the cases. The current models predict within 2 percent of the actual length in three cases; within 15 percent in the fourth case. The IGSHPA method predicts lengths anywhere from 20 to 130 percent longer than the actual length. The fact that the actual lengths and predicted lengths compare very favourably testifies to the accuracy of the sizing undertaken by the original investigators [6].

3.3 Results of Comparisons - Energy Analysis

This section will present results of comparisons of predicted energy use and seasonal efficiency, for both the NWWA models and the ASHRAE or IGSHPA methods. The measured house loads are used, together with specifications for the actual heat pumps used in each case.

Tables 6, 7 and 8 compare the energy performance predicted for cases #2, #3 and #4 using the **actual** length of the ground-coil from each case. An analyst would use this approach, after installation, to predict system energy use. In Tables 9,10 and 11, the **minimum EWT** is known but the length has not been established. The designer or analyst simply wants to examine the implication of the minimum EWT on seasonal performance.

The actual choice of minimum entering liquid or water temperature, EWT, can impact on both performance and cost of a GSHP installation. Too low a minimum EWT will result in lower COPs and hence higher operating costs. With too high a minimum EWT, while COPs will be higher, the heat exchanger length will be excessive raising the installed cost of the installation. The designer needs to strike a balance between operating and installation cost and find a value somewhere in between the two extremes.

The differences in the HSPF (Heating Seasonal Performance Factor), defined as heating load met by the system divided by total system electrical energy use, including supplementary energy, are no greater than 10 percent in either the **actual** length case or the **minimum EWT** case. The difference in cooling season efficiency, SEER (Seasonal Energy Efficiency Ratio), defined as cooling load met by the system divided by the total system electrical energy use is also no greater than 10 percent in the **minimum EWT** than in the **actual** length case.

In Table 12, the results of another comparison are presented. Here, the case #3 house load has been increased by about 40 percent. The heat pump capacity remains the same. The **actual** heat exchanger length is used. The purpose of this example was to examine the effect of equipment undersizing given by a lower value of the ratio, heat pump capacity to house design load, on each method's predicted seasonal performance. With the higher house load, the IGSHPA method appears to predict greater energy use than before (i.e. the difference goes from 4 percent to 6 percent). However, the difference is small.

4. CONCLUSIONS

The horizontal models, with soil moisture freezing effect, compared favourably here to real data. The accuracy of minimum entering water temperature would suggest that design lengths based on these algorithms will not be excessive. The vertical heat exchanger comparison with real data is good as well. The lake-loop model could not be validated as no suitable data set could be identified.

Monthly time steps were employed in three of the four cases, while a semi-monthly time step was employed in the single pipe horizontal model. The monthly time step should be sufficiently accurate for the residential models and ideally suited to simplified energy analysis methods based on the bin method.

One can conclude that the length prediction inaccuracy associated with currently used models has little effect on predicted seasonal performance (HSPF or SEER) of a ground-source heat pump. Even in the case where the heat pump met only 70 percent of the design heating load, which is the lowest recommended capacity called for in the CSA Design and Installation Standard, CSA-C445, the energy use predictions of the NWWA and ASHRAE/IGSHPA methods compared favourably.

The most important effect of oversizing of the heat exchanger is the significant increase in installed cost and the degraded customer cost/benefit. By eliminating the apparent significant oversizing associated with the IGSHPA method, which is the sizing methodology used in most proprietary models, the new models will help to ensure maximum benefit from the Ontario Hydro incentive for the customer. By lowering the installed cost, the new prediction models should broaden the potential market for GSHPs in Canada.

This investigation also concluded that while the energy analysis capability for GSHP systems was important, a design tool for heat exchanger sizing was more important to the industry. At the present time, the Hot 2000™ program is capable only of the energy analysis. The need would appear to exist for a stand-alone package, which could use loads and other data imported from another program, such as Hot™ 2000. This stand-alone design program when developed and validated could be referenced in the Design and Installation Standard for Ground-Source Heat Pumps, CSA C445. The latter is currently mandatory for all GSHP installation incentives from Ontario Hydro. At the present time, the Standard references the sizing method in the ASHRAE Design/Data Manual [3] which has been shown in this project to significantly oversize heat exchangers.

5. RECOMMENDATIONS

A future project should assemble appropriate soil temperature data for Canadian locations. The data needed are: the normal annual undisturbed mean soil temperature; the amplitude of the soil surface temperature and the phase shift constant (i.e. the time between the minimum air temperature and minimum undisturbed mean ground temperature). Data for Canadian locations could be developed from soil temperature data recorded at various sites across Canada. As an alternative, a technique described in "Development of a Soil Temperature Prediction Model" prepared by G. Moore, for the R2000 Home Program, EMR in 1986, could provide the necessary soil temperature data. This technique is embodied in Hot™ 2000 in the existing ground source heat pump model and should be examined as an alternative to the use of actual soil temperature measurements. Validation of the model against real soil temperature data is recommended prior to implementation.

Additional work should be undertaken to establish the appropriate time-step for EWT calculation in commercial applications, where the energy analysis may need to be performed on an hourly basis. Applications with simultaneous heating and cooling will likely require a different time-step to calculate the EWT. The existing model accounts for past history of soil temperature distribution and if the calculations were performed on an hourly rather than monthly basis, memory requirements to store the calculations would become extensive. Comparisons with monitored data from a commercial building application would permit an evaluation of the time-step requirements.

It is recommended that the stand-alone, heat exchanger sizing program development be undertaken, compatible with Hot 2000™ as the source for design loads and other data. Organizations other than EMR may be interested in co-funding such a development to

provide a standardized method for GSHP heat exchanger sizing.

This project resulted in a model for lake-loop heat exchanger systems. The absence of real data prevented determining the accuracy of the model. A project could monitor the performance of an in-situ lake-loop heat pump system to obtain the data necessary to check the accuracy of the model. Approximately 10 to 15 percent of all closed-loop systems installed in Canada are lake-loop-designs.

A model for horizontal spiral heat exchanger ground source heat pumps has not been developed here. This heat exchanger is becoming more popular and a predictive tool for sizing and energy analysis is required. A closed-form solution to the equations governing heat transfer to and from a spiral heat exchanger is not possible. It is recommended that sufficient operating data be gathered on spiral heat exchangers, at different sites, preferably under controlled conditions, to enable the development of an empirical model. This could simply involve modification of one of the existing horizontal heat exchanger models, but would be correlated or validated against real data.

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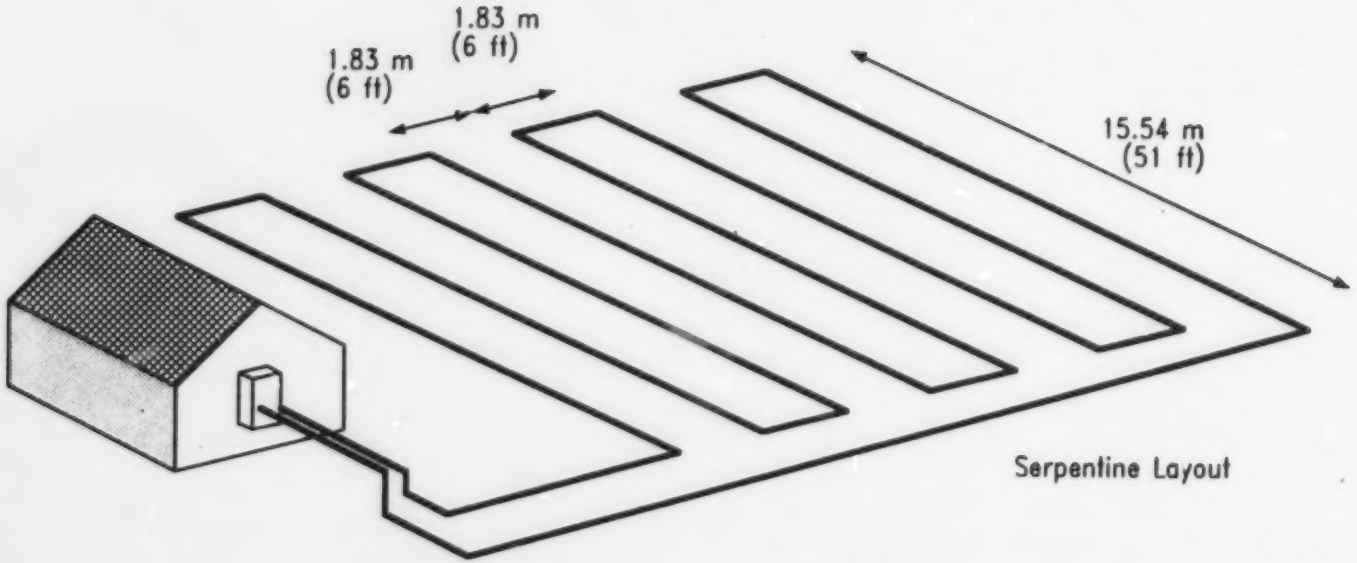
Table 1 Model Validation #1 - Parameter Values for Comparison Run

Single Pipe Horizontal

Design Characteristics			
Winter Design Temperature		- 12.0 °C	(10.4 °F)
Winter Thermostat Setting		21.11 °C	(70 °F)
Design Heating Load		N/A (HP Run-Time Provided)	
Piping Characteristics			
Piping Type		Polyethylene SCH-40 - nominal 1.5 in.	
Outside Pipe Diameter		0.046 m	(1.830 in)
Inside Pipe Diameter		0.041 m	(1.610 in)
Pipe Thermal Conductivity		0.460 W/m.°C	(0.266 Btu/h.ft.°F)
Coil Layout Characteristics			
Pipe Burial Depth		1.219 m	(4 ft)
Number of Trenches		N/A (Serpentine)	
Total Piping Length		154.5 m	(507 ft)
Soil Characteristics			
Soil Annual Mean Temperature		12.2 °C	(54 °F)
Amplitude of Surface Soil Temperature		15.6 °C	(28 °F)
Phase Constant		20 days	-
Soil Thermal Diffusivity,	Winter	1.0 10-6 m2/s	(0.039 ft2/h)
	Summer	1.0 10-6 m2/s	(0.039 ft2/h)
Soil Thermal Conductivity,	Winter	1.731 W/m.°C	(1.00 Btu/h.ft.°F)
	Summer	1.731 W/m.°C	(1.00 Btu/h.ft.°F)
Heat Pump Characteristics			
Manufacturer & Model		Command Aire - SWPR 261 EA	
Nominal Size		2 ton	-
Flow Rate		0.252 kg/s	(4 gpm)



Total Pipe Length : 154.5 m (507 ft)

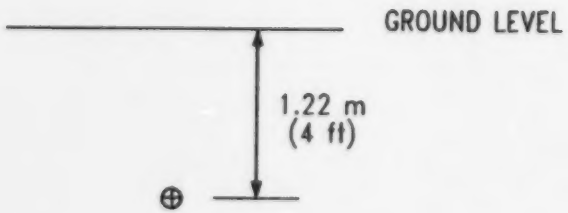


Figure 1 Model Validation #1 – Earth Loop Configuration : Single Pipe Horizontal

Figure 2 Model Validation #1 - Hor. Single Pipe
Measured vs. Predicted Entering Water Temperature

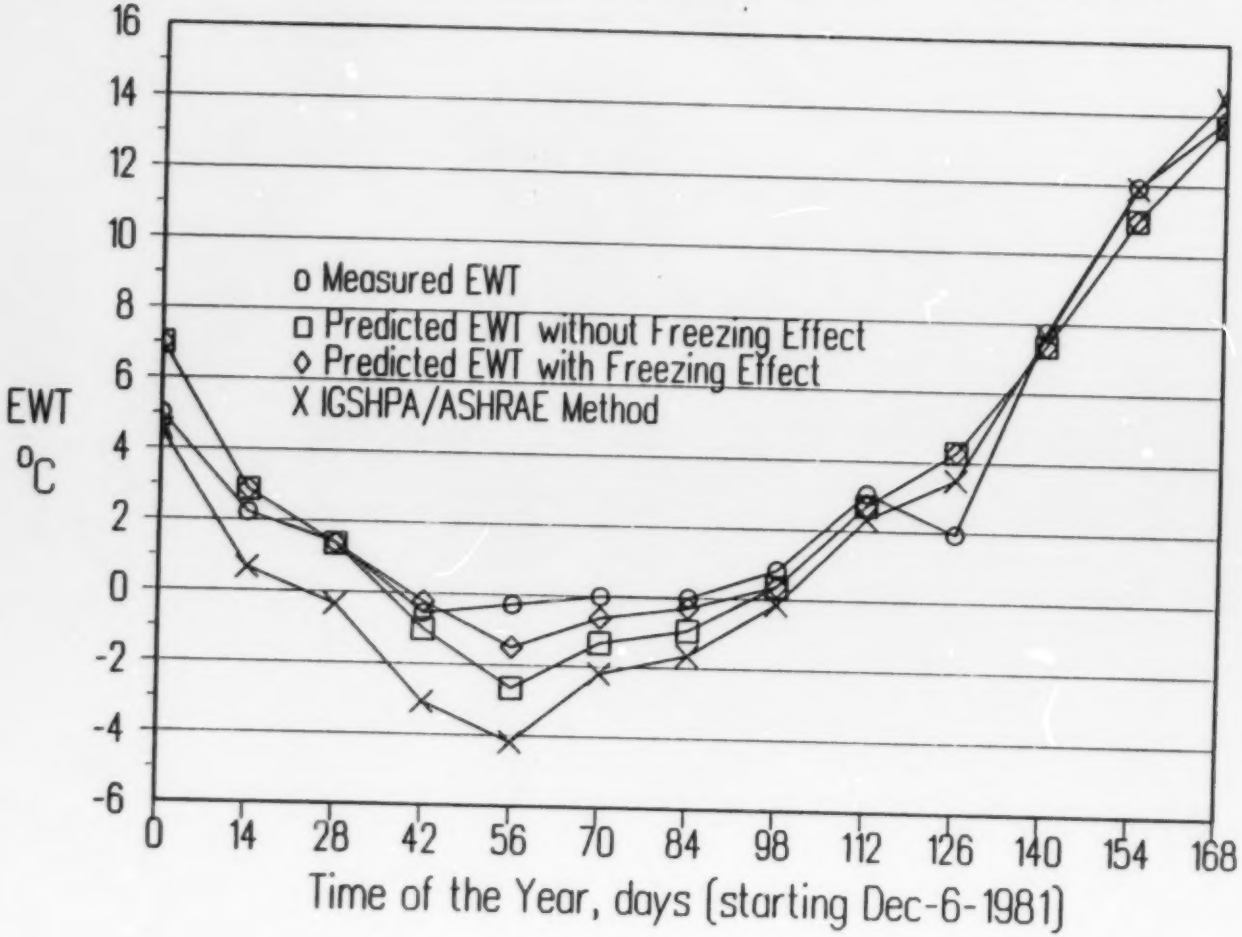


Table 2 Model Validation #2 - Parameter Values for Comparison Run

Two Pipe Over/Under Horizontal

Design Characteristics			
Winter Design Temperature		- 14.45 °C	(6 °F)
Winter Thermostat Setting		21.11 °C	(70 °F)
Design Heating Load		9.021 kW	(30,801 Btu/h)
Summer Design Temperature		29.45 °C	(85 °F)
Summer Thermostat Setting		23.89 °C	(75 °F)
Design Cooling Load		4.097 kW	(13,989 Btu/h)
Piping Characteristics			
Piping Type		Polybutylene SDR-17 - nominal 1 in.	
Outside Pipe Diameter		0.033 m	(1.315 in)
Inside Pipe Diameter		0.029 m	(1.160 in)
Pipe Thermal Conductivity		0.216 W/m.°C	(0.125 Btu/h.ft.°F)
Coil Layout Characteristics			
Top Pipe Burial Depth		0.914 m	(3 ft)
Bottom Pipe Burial Depth		1.524 m	(5 ft)
Number of Trenches		6	-
Trench Length		21.56 m	(70.75 ft)
Total Piping Length		320 m	(1050 ft)
Soil Characteristics			
Soil Annual Mean Temperature		10 °C	(50 °F)
Amplitude of Surface Soil Temperature		13.3 °C	(24 °F)
Phase Constant		38 days	-
Soil Thermal Diffusivity,	Winter	6.42 10-7 m2/s	(0.025 ft2/h)
	Summer	5.79 10-7 m2/s	(0.0225 ft2/h)
Soil Thermal Conductivity,	Winter	1.297 W/m.°C	(0.75 Btu/h.ft.°F)
	Summer	1.081 W/m.°C	(0.625 Btu/h.ft.°F)
Heat Pump Characteristics			
Manufacturer & Model		Command Aire - SWPR 261 EA	
Nominal Size		2 ton	-
Flow Rate		0.516 kg/s	(8.2 gpm)

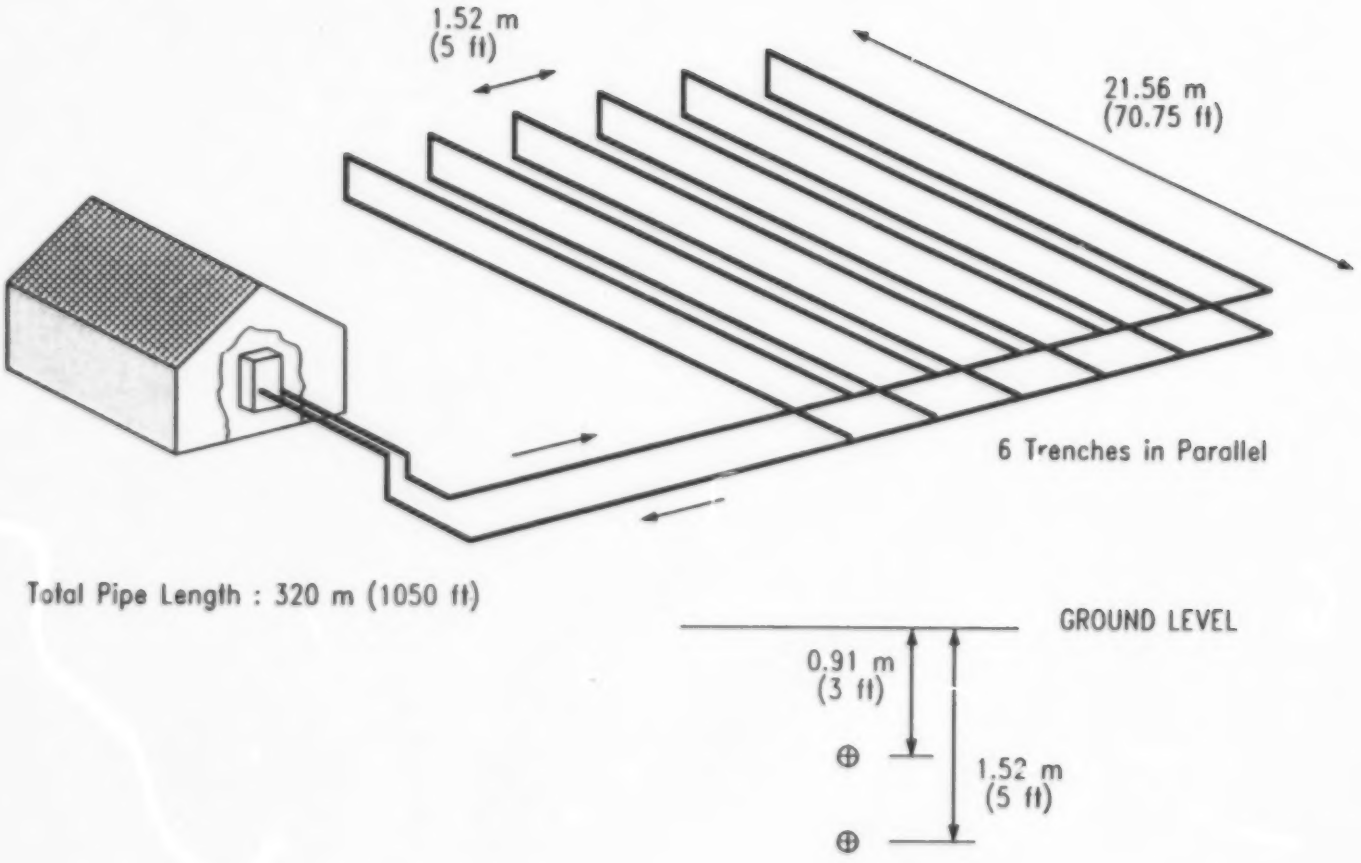


Figure 3 Model Validation #2 - Earth Loop Configuration : Two Pipe Over/Under Horizontal

Figure 4 Model Validation #2 - Hor. Two Pipes
Measured vs. Predicted Entering Water Temperature

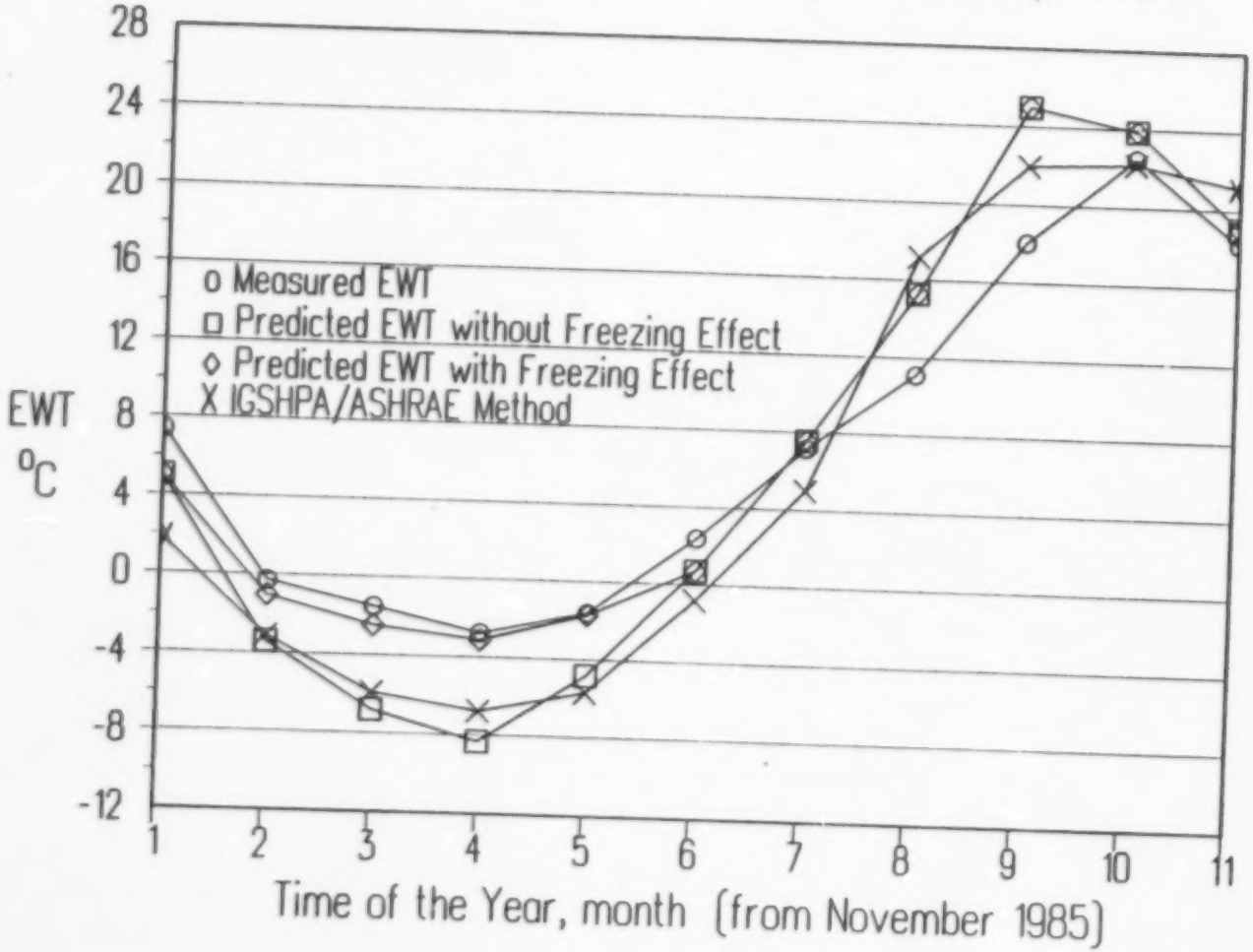


Table 3 Model Validation #3 - Parameter Values for Comparison Run

Two Pipe Side-By-Side Horizontal

Design Characteristics			
Winter Design Temperature		- 16.67 °C	(2 °F)
Winter Thermostat Setting		21.11 °C	(70 °F)
Design Heating Load		8.107 kW	(27,681 Btu/h)
Summer Design Temperature		31.39 °C	(88.5 °F)
Summer Thermostat Setting		23.89 °C	(75 °F)
Design Cooling Load		4.503 kW	(15,375 Btu/h)
Piping Characteristics			
Piping Type		Polybutylene SDR-17 - nominal 1 in.	
Outside Pipe Diameter		0.033 m	(1.315 in)
Inside Pipe Diameter		0.029 m	(1.160 in)
Pipe Thermal Conductivity		0.216 W/m.°C	(0.125 Btu/h.ft.°F)
Coil Layout Characteristics			
Pipe Burial Depth		0.914 m	(3 ft)
Space Between Pipes in Trench		0.61 m	(2 ft)
Number of Trenches		N/A (Serpentine)	
Total Piping Length		396.2 m	(1300 ft)
Soil Characteristics			
Soil Annual Mean Temperature		10 °C	(50 °F)
Amplitude of Surface Soil Temperature		13.3 °C	(24 °F)
Phase Constant		38 days	-
Soil Thermal Diffusivity,	Winter	6.42 10-7 m2/s	(0.025 ft2/h)
	Summer	5.79 10-7 m2/s	(0.0225 ft2/h)
Soil Thermal Conductivity,	Winter	1.297 W/m.°C	(0.75 Btu/h.ft.°F)
	Summer	1.081 W/m.°C	(0.625 Btu/h.ft.°F)
Heat Pump Characteristics			
Manufacturer & Model		Cantherm - DUO 520	
Nominal Size		4.5 ton	-
Flow Rate		0.755 kg/s	(12 gpm)

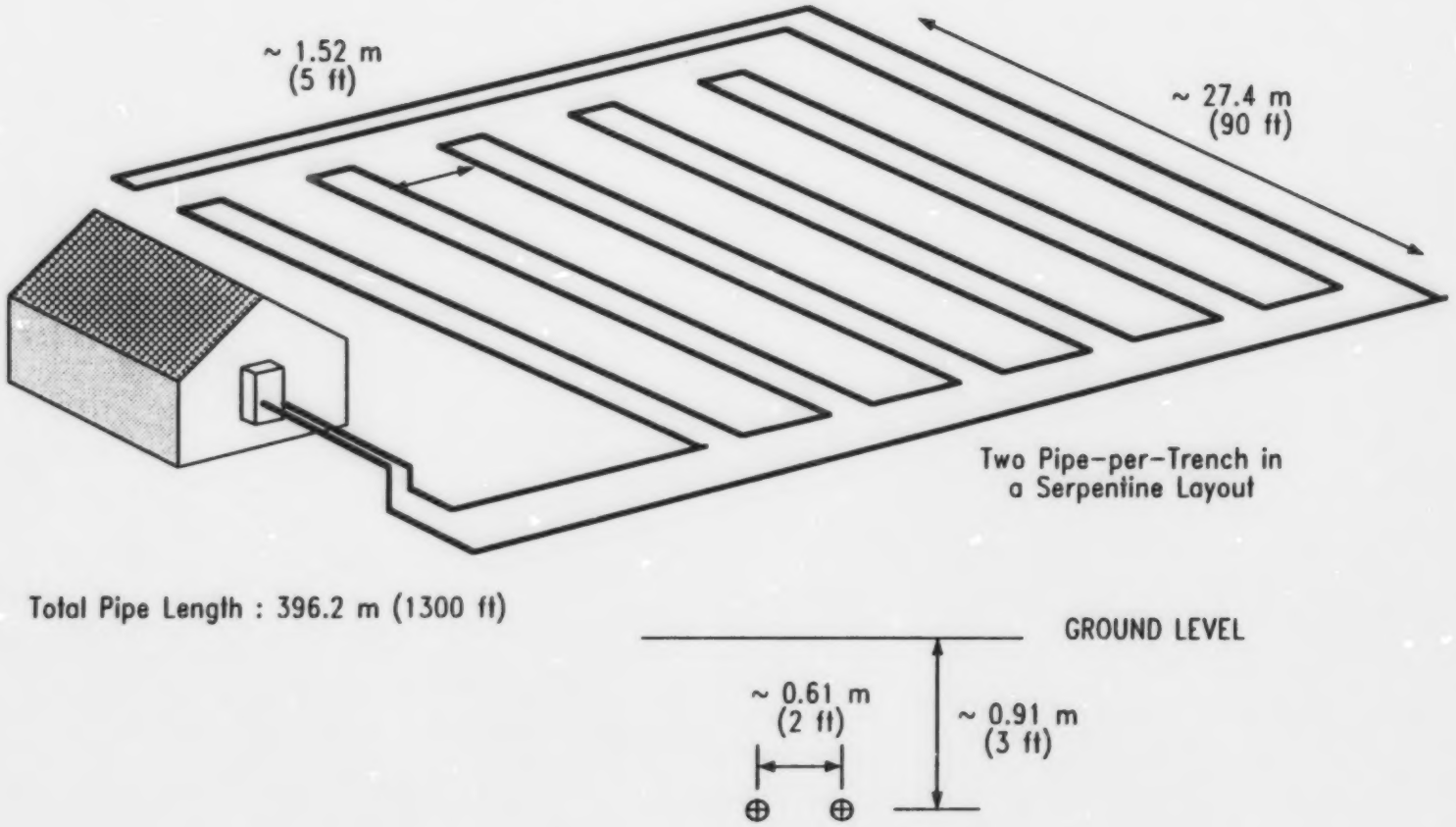


Figure 5 Model Validation #3 - Earth Loop Configuration : Two Pipe Side-By-Side Horizontal

Figure 6 Model Validation #3 - Hor. Two Pipes
Measured vs. Predicted Entering Water Temperature

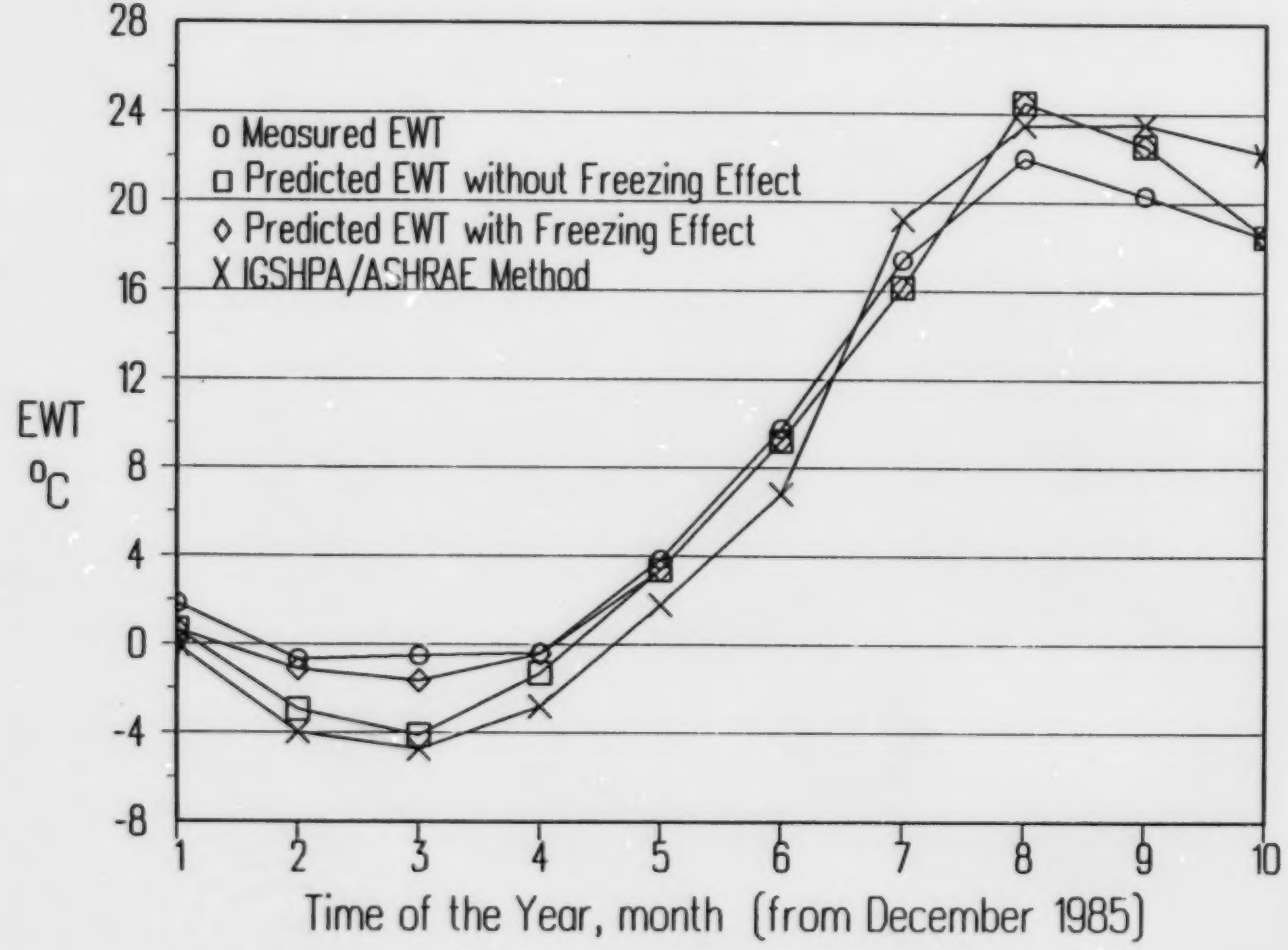


Table 4 Model Validation #4 - Parameter Values for Comparison Run

3 U-Tube Parallel Vertical

Design Characteristics		
Winter Design Temperature	- 14.45 °C	(6 °F)
Winter Thermostat Setting	21.11 °C	(70 °F)
Design Heating Load	16.15 kW	(55,143 Btu/h)
Summer Design Temperature	29.45 °C	(85 °F)
Summer Thermostat Setting	23.89 °C	(75 °F)
Design Cooling Load	5.74 kW	(19,599 Btu/h)
Piping Characteristics		
Piping Type	Polybutylene SDR-17 - nominal 1 in.	
Outside Pipe Diameter	0.033 m	(1.315 in)
Inside Pipe Diameter	0.029 m	(1.160 in)
Pipe Thermal Conductivity	0.216 W/m.°C	(0.125 Btu/h.ft.°F)
Coil Layout Characteristics		
Borehole Length	45.72 m	(150 ft)
Number of Borehole	3	-
Distance Between Boreholes	3.05 m, 6.64 m	(10 ft, 22 ft)
Total Piping Length	274.3 m	(900 ft)
Soil Characteristics		
Soil Annual Mean Temperature	10 °C	(50 °F)
Amplitude of Surface Soil Temperature	13.3 °C	(24 °F)
Phase Constant	38 days	-
Soil Thermal Diffusivity, Winter	6.42 10-7 m2/s	(0.025 ft2/h)
Summer	5.79 10-7 m2/s	(0.0225 ft2/h)
Soil Thermal Conductivity, Winter	1.297 W/m.°C	(0.75 Btu/h.ft.°F)
Summer	1.081 W/m.°C	(0.625 Btu/h.ft.°F)
Rock Thermal Diffusivity, Winter/Summer	1.29 10-6 m2/s	(0.050 ft2/h)
Rock Thermal Conductivity, Winter/Summer	3.459 W/m.°C	(2.00 Btu/h.ft.°F)
Heat Pump Characteristics		
Manufacturer & Model	Command Aire - SWPR 351 EA	
Nominal Size	3 ton	-
Flow Rate	0.598 kg/s	(9.5 gpm)

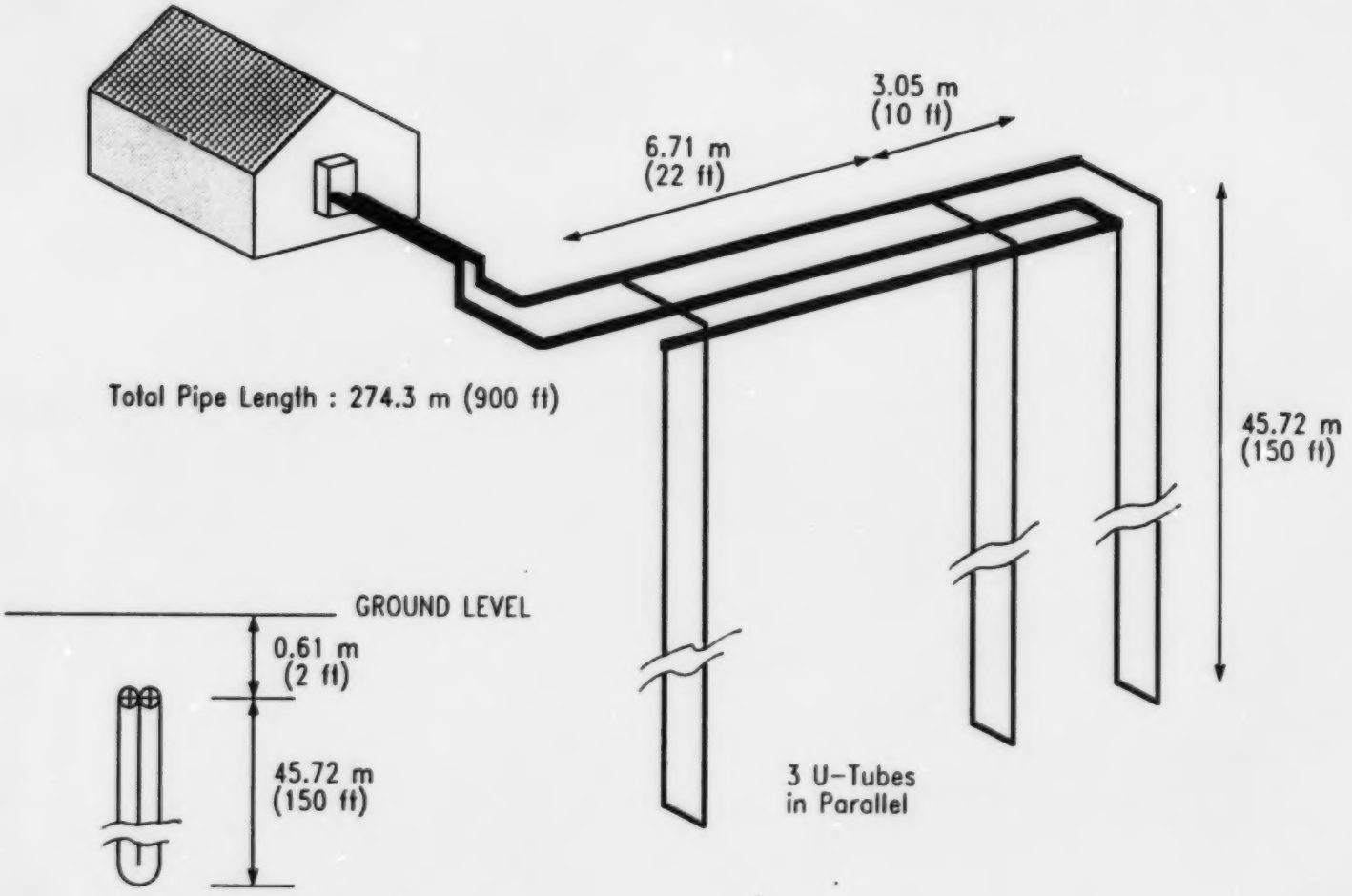


Figure 7 Model Validation #4 – Earth Loop Configuration : 3 U-Tubes Parallel Vertical

Figure 8 Model Validation #4 - Vert. 3 U-Tubes
Measured vs. Predicted Entering Water Temperature

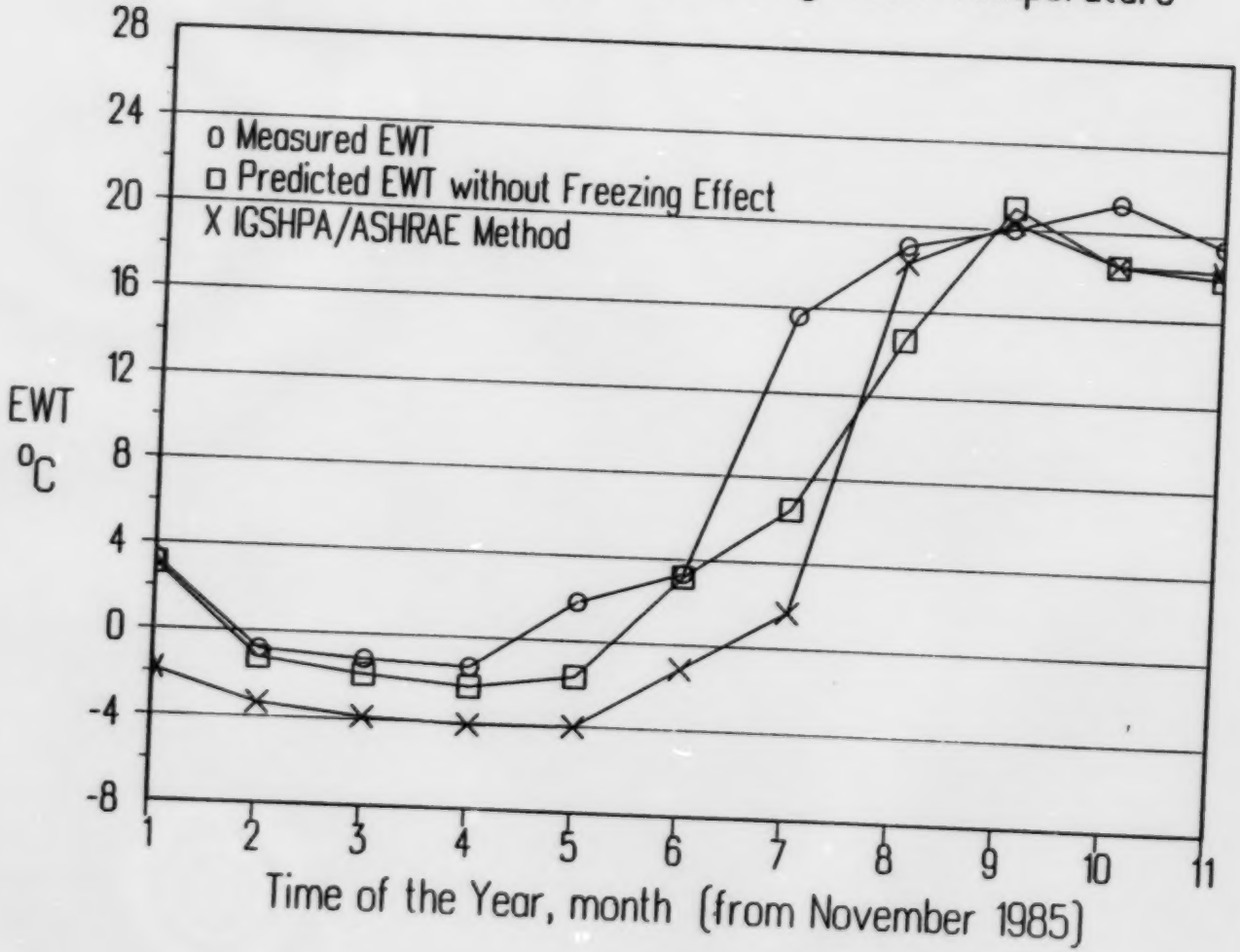


Table 5 Model Validation - Comparison Runs

Total Pipe Length Predicted vs. Actual

Configuration	Actual Length m	(ft)	Predicted Length* m	(ft)	Difference %
#1 Horizontal, Single pipe-per-trench Serpentine	154.6	507	A: 152.0 B: 280.6 C: 259.1	499 921 850	-1.7 81.5 67.6
#2 Horizontal, Two pipe-per-trench (Over/Under) 6 Trenches in Parallel	320.0	1050	A: 278.0 B: 545.0 C: 310.9	912 1788 1020	-13.1 70.3 -2.8
#3 Horizontal, Two pipe-per-trench (Side-by-side) Serpentine	396.2	1300	A: 398.0 B: 909.0 C: 699.5	1306 2982 2295	0.5 129.4 76.6
#4 Vertical, Single U-tube-per-borehole 3 Boreholes in Parallel	274.3	900	A: 271.7 B: 330.0 C: 365.8	891 1083 1200	-0.9 20.3 33.4

* Length predicted at the lowest measured entering water temperature (EWT) within 1.0 °C accuracy :

- A: prediction from the NWWA length calculation procedure
- B: prediction from the IGSHPA length calculation procedure
- C: prediction from the ASHRAE Manual 'Rule of Thumb'

TABLE 6 MODEL VALIDATION #2 - ENERGY ANALYSIS COMPARISON
HORIZONTAL SYSTEM - 2 PIPE-PER-TRENCH OVER/UNDER

NWWA METHOD : PREDICTION WITH ACTUAL LENGTH

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	12.9	8996	3.72		173	1135	378	39	1552	0	
Nov	5.0	7593	3.34		410	2179	839	92	3113	3	
Dec	-1.0	6416	3.04		596	2569	1124	134	3993	166	
Jan	-2.5	6125	2.97		639	2596	1173	144	4468	556	
Feb	-3.1	5994	2.94		582	2301	1055	131	4004	517	
Mar	-1.7	6281	3.01		563	2359	1047	127	3616	83	
Apr	0.7	6762	3.13		313	1439	606	70	2115	0	
May	7.6	8063	3.46		157	899	330	35	1264	0	HSPF
HEATING SEASON :					3431	15477	6551	772	24125	1325	2.79
Jun	15.3	8350	4.18	14.27	23	241	41	5	195		
Jul	25.0	7695	3.46	11.82	73	725	146	16	563		
Aug	23.9	7770	3.54	12.10	30	301	59	7	235		
Sep	19.1	8096	3.90	13.32	24	244	44	5	194	SEER	
COOLING SEASON :					151	1511	291	34	1187	12.47	

IGSHPA METHOD : PREDICTION WITH ACTUAL LENGTH

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	9.2	8358	3.54		186	1113	396	42	1552	2	
Nov	1.9	6995	3.19		442	2121	871	99	3113	22	
Dec	-3.1	5996	2.94		623	2466	1130	140	3993	257	
Jan	-5.9	5414	2.80		674	2348	1150	152	4468	818	
Feb	-6.8	5233	2.76		612	2044	1022	138	4004	800	
Mar	-5.7	5456	2.81		622	2186	1065	140	3616	225	
Apr	-0.8	6463	3.05		327	1422	619	74	2115	1	
May	5.0	7588	3.34		167	885	341	37	1264	0	HSPF
HEATING SEASON :					3652	14584	6594	822	24125	2125	2.53
Jun	17.0	8236	4.08	13.85	24	243	43	5	195		
Jul	22.2	7885	3.67	12.53	71	716	137	16	563		
Aug	22.0	7898	3.69	12.58	30	298	57	7	235		
Sep	21.1	7959	3.75	12.81	24	246	46	5	194	SEER	
COOLING SEASON :					149	1504	283	34	1187	12.78	

SUMMARY

	NWWA	IGSHPA	DIFFERENCE
HEATING ENERGY :	8648	9541	-893 kWh
COOLING ENERGY :	325	317	8 kWh
TOTAL ENERGY :	8973	9858	-885 kWh

TABLE 7 MODEL VALIDATION #3 - ENERGY ANALYSIS COMPARISON
HORIZONTAL SYSTEM - 2 PIPE-PER-TRENCH SIDE-BY-SIDE

NWWA METHOD : PREDICTION WITH ACTUAL LENGTH

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	13.7	12675	2.36		113	826	556	51	1432	0	
Nov	7.1	10939	2.15		205	1202	949	92	2244	0	
Dec	0.6	9220	1.95		368	1651	1572	165	3389	0	
Jan	-1.1	8751	1.89		476	1969	1987	214	4187	18	
Feb	-1.7	8613	1.88		410	1652	1697	185	3545	11	
Mar	-0.4	8933	1.92		344	1470	1450	155	3075	0	
Apr	3.4	9943	2.04		166	841	737	75	1653	0	
May	9.2	11491	2.22		73	460	344	33	836	0	HSPF
HEATING SEASON :					2156	10071	9292	970	20362	30	1.98
Jun	16.2	13874	2.75	9.38	16	296	72	7	217		
Jul	24.5	12841	2.37	8.09	34	621	169	15	385		
Aug	22.6	13079	2.46	8.39	26	483	128	12	343		
Sep	18.5	13584	2.64	9.02	9	167	42	4	121	SEER	
COOLING SEASON :					85	1566	410	38	1067	8.12	

IGSHPA METHOD : PREDICTION WITH ACTUAL LENGTH

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	10.9	11939	2.27		120	802	576	54	1432	0	
Nov	5.5	10515	2.10		213	1178	970	96	2244	0	
Dec	-0.1	9037	1.93		375	1631	1588	169	3389	0	
Jan	-4.0	7998	1.80		519	1850	2064	233	4187	40	
Feb	-4.7	7797	1.78		450	1539	1768	203	3545	35	
Mar	-2.8	8305	1.84		370	1405	1503	167	3075	0	
Apr	1.8	9522	1.99		174	321	754	78	1653	0	
May	6.8	10852	2.14		77	446	356	35	836	0	HSPF
HEATING SEASON :					1888	9673	9581	1034	20362	74	1.90
Jun	19.2	13493	2.61	8.91	16	296	75	7	217		
Jul	23.5	12967	2.42	8.25	33	614	165	15	385		
Aug	24.4	12851	2.37	8.10	27	489	133	12	343		
Sep	22.5	13089	2.46	8.40	9	172	46	4	121	SEER	
COOLING SEASON :					85	1571	418	38	1067	7.98	

SUMMARY

	NWWA	IGSHPA	DIFFERENCE
HEATING ENERGY :	10291	10689	-398 kWh
COOLING ENERGY :	448	456	-8 kWh
TOTAL ENERGY :	10740	11146	-406 kWh

TABLE 8 MODEL VALIDATION #4 - ENERGY ANALYSIS COMPARISON
VERTICAL SYSTEM - 3 U-TUBES IN PARALLEL

NWWA METHOD : PREDICTION WITH ACTUAL LENGTH

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	7.6	11472	3.20		242	1910	760	109	2779	0	
Nov	3.1	10282	2.96		522	3553	1574	235	5573	211	
Dec	-1.2	9083	2.74		675	3893	1935	304	7149	1017	
Jan	-1.9	8891	2.70		695	3894	1972	313	7999	1820	
Feb	-2.3	8769	2.68		626	3440	1763	281	7168	1683	
Mar	-1.7	8957	2.72		650	3679	1851	293	6473	650	
Apr	3.2	10307	2.97		366	2501	1106	165	3786	15	
May	6.5	11202	3.14		202	1544	629	91	2264	0	HSPF
HEATING SEASON :					3977	24415	11590	1790	43190	5395	2.30
Jun	14.7	12852	3.97	13.53	21	341	59	10	273		
Jul	21.0	11962	3.50	11.93	66	1014	196	30	789		
Aug	18.4	12323	3.69	12.58	27	418	77	12	329		
Sep	18.0	12384	3.72	12.69	22	346	63	10	272	SEER	
COOLING SEASON :					136	2119	396	61	1662	12.42	

IGSHPA METHOD : PREDICTION WITH ACTUAL LENGTH

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	2.3	10077	2.92		276	1828	826	124	2779	0	
Nov	-1.8	8905	2.71		572	3211	1624	257	5573	481	
Dec	-3.3	8470	2.63		692	3634	1919	312	7149	1284	
Jan	-3.9	8320	2.60		708	3627	1945	319	7999	2108	
Feb	-4.1	8256	2.59		635	3219	1738	286	7168	1926	
Mar	-4.0	8282	2.60		671	3415	1839	302	6473	917	
Apr	-1.1	9108	2.74		406	2353	1166	183	3786	85	
May	1.7	9901	2.89		229	1481	680	103	2264	0	HSPF
HEATING SEASON :					4189	22787	11737	1885	43190	6802	2.11
Jun	18.3	12346	3.70	12.62	22	347	64	10	273		
Jul	20.3	12062	3.55	12.11	65	1011	193	29	789		
Aug	18.5	12316	3.68	12.57	27	418	77	12	329		
Sep	18.3	12339	3.69	12.61	22	347	64	10	272	SEER	
COOLING SEASON :					136	2122	398	61	1662	12.35	

SUMMARY

	NWWA	IGSHPA	DIFFERENCE
HEATING ENERGY :	18775	20423	-1648 kWh
COOLING ENERGY :	457	459	-2 kWh
TOTAL ENERGY :	19232	20883	-1651 kWh

TABLE 9 MODEL VALIDATION #2 - ENERGY ANALYSIS COMPARISON
HORIZONTAL SYSTEM - 2 PIPE-PER-TRENCH OVER/UNDER

NWWA METHOD : PREDICTION WITH ACTUAL MINIMUM EWT

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	12.6	8953	3.71		173	1134	379	39	1552	0	
Nov	4.2	7437	3.30		418	2166	848	94	3113	5	
Dec	-1.7	6292	3.01		604	2539	1126	136	3993	193	
Jan	-3.1	5996	2.94		645	2553	1170	145	4468	599	
Feb	-3.8	5856	2.91		588	2259	1052	132	4004	560	
Mar	-2.2	6190	2.99		569	2344	1051	128	3616	93	
Apr	0.4	6691	3.11		316	1435	609	71	2115	0	
May	7.3	8014	3.45		158	898	331	35	1264	0	HSPF
HEATING SEASON :					3471	15327	6566	781	24125	1450	2.74
Jun	15.6	8331	4.16	14.20	23	242	42	5	195		
Jul	26.3	7607	3.37	11.49	74	730	151	17	563		
Aug	24.8	7709	3.48	11.87	30	302	61	7	235		
Sep	19.3	8081	3.89	13.26	24	244	45	5	194	SEER	
COOLING SEASON :					152	1518	297	34	1187	12.22	

IGSHPA METHOD : PREDICTION WITH ACTUAL MINIMUM EWT

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	11.3	8728	3.65		178	1126	385	40	1552	0	
Nov	5.2	7625	3.35		408	2180	837	92	3113	4	
Dec	0.3	6681	3.11		578	2619	1113	130	3993	131	
Jan	-2.7	6078	2.96		641	2582	1172	144	4468	569	
Feb	-3.8	5856	2.91		588	2259	1052	132	4004	560	
Mar	-2.6	6099	2.97		576	2330	1056	130	3616	101	
Apr	1.7	6958	3.18		304	1451	598	68	2115	-3	
May	6.8	7922	3.43		160	895	333	36	1264	0	HSPF
HEATING SEASON :					3433	15443	6546	772	24125	1383	2.78
Jun	15.7	8324	4.15	14.17	23	242	42	5	195		
Jul	19.9	8040	3.84	13.11	70	709	131	16	563		
Aug	20.4	8006	3.80	12.98	29	296	55	7	235		
Sep	19.5	8067	3.87	13.21	24	245	45	5	194	SEER	
COOLING SEASON :					147	1492	272	33	1187	13.27	

SUMMARY

	NWWA	IGSHPA	DIFFERENCE
HEATING ENERGY :	8798	8682	116 kWh
COOLING ENERGY :	331	305	26 kWh
TOTAL ENERGY :	9129	8987	142 kWh

TABLE 10 MODEL VALIDATION #3 - ENERGY ANALYSIS COMPARISON
HORIZONTAL SYSTEM - 2 PIPE-PER-TRENCH SIDE-BY-SIDE

NWWA METHOD : PREDICTION WITH ACTUAL MINIMUM EWT

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	13.7	12675	2.36		113	826	556	51	1432	0	
Nov	7.2	10945	2.16		205	1203	949	92	2244	0	
Dec	0.7	9225	1.95		367	1652	1572	165	3389	0	
Jan	-1.1	8756	1.90		476	1970	1987	214	4187	16	
Feb	-1.7	8607	1.88		411	1651	1697	185	3545	12	
Mar	-0.4	8936	1.92		344	1471	1450	155	3075	0	
Apr	3.4	9946	2.04		166	841	737	75	1653	0	
May	9.2	11491	2.22		73	460	344	33	836	0	HSPF
HEATING SEASON :					2156	10073	9292	970	20362	27	1.98
Jun	16.2	13875	2.75	9.39	16	295	72	7	217		
Jul	24.5	12846	2.37	8.10	34	621	169	15	385		
Aug	22.6	13082	2.46	8.39	26	482	128	12	343		
Sep	18.5	13585	2.64	9.02	9	167	42	4	121	SEER	
COOLING SEASON :					85	1585	410	38	1067	8.13	

IGSHPA METHOD : PREDICTION WITH ACTUAL MINIMUM EWT

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	12.9	12463	2.34		115	819	561	52	1432	0	
Nov	8.0	11173	2.18		201	1215	938	90	2244	0	
Dec	2.9	9829	2.02		345	1713	1520	155	3389	0	
Jan	-0.7	8862	1.91		471	1985	1976	212	4187	14	
Feb	-1.7	8607	1.88		411	1651	1697	185	3545	12	
Mar	-0.2	9010	1.93		341	1478	1444	154	3075	0	
Apr	3.7	10031	2.05		165	845	734	74	1653	0	
May	8.4	11271	2.19		74	455	348	33	836	0	HSPF
HEATING SEASON :					1732	10163	9217	955	20362	27	2.00
Jun	16.3	13852	2.74	9.36	16	296	72	7	217		
Jul	20.1	13384	2.57	8.77	33	608	156	15	385		
Aug	21.3	13239	2.52	8.59	26	480	125	12	343		
Sep	19.9	13415	2.58	8.81	9	168	43	4	121	SEER	
COOLING SEASON :					83	1552	395	37	1067	8.42	

SUMMARY

	NWWA	IGSHPA	DIFFERENCE
HEATING ENERGY :	10289	10199	90 kWh
COOLING ENERGY :	448	433	15 kWh
TOTAL ENERGY :	10737	10632	105 kWh

TABLE 11 MODEL VALIDATION #4 - ENERGY ANALYSIS COMPARISON
VERTICAL SYSTEM - 3 U-TUBES IN PARALLEL

NWWA METHOD : PREDICTION WITH ACTUAL MINIMUM EWT

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	7.6	11467	3.20		242	1909	760	109	2779	0	
Nov	3.0	10260	2.96		522	3548	1693	118	5573	215	
Dec	-1.3	9060	2.74		676	3884	2086	152	7149	1026	
Jan	-2.0	8868	2.70		696	3884	2128	157	7999	1831	
Feb	-2.4	8746	2.68		626	3431	1903	141	7168	1693	
Mar	-1.8	8925	2.71		651	3668	1998	147	6473	661	
Apr	3.1	10288	2.96		367	2499	1189	82	3786	16	
May	6.5	11194	3.14		202	1543	675	45	2264	0	HSPF
HEATING SEASON :					3982	24366	12432	950	43190	5442	2.29
Jun	14.7	12845	3.96	13.52	21	342	64	5	273		
Jul	21.1	11945	3.49	11.90	66	1015	211	15	789		
Aug	18.5	12310	3.68	12.56	27	418	83	6	329		
Sep	18.1	12373	3.71	12.67	22	346	68	5	272	SEER	
COOLING SEASON :					136	2120	427	31	1662	12.40	

IGSHPA METHOD : PREDICTION WITH ACTUAL MINIMUM EWT

MONTH	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	3.5	10396	2.99		267	1848	810	120	2779	0	
Nov	-0.2	9364	2.79		556	3339	1739	125	5573	370	
Dec	-1.7	8957	2.72		679	3841	2085	153	7149	1070	
Jan	-2.2	8807	2.69		697	3857	2126	157	7999	1859	
Feb	-2.4	8746	2.68		626	3431	1903	141	7168	1693	
Mar	-2.3	8787	2.69		655	3614	1996	147	6473	719	
Apr	0.5	9561	2.83		391	2414	1233	88	3786	51	
May	3.0	10247	2.96		221	1498	716	50	2264	0	HSPF
HEATING SEASON :					4092	23842	12609	981	43190	5758	2.23
Jun	16.9	12537	3.80	12.97	22	345	67	5	273		
Jul	18.6	12299	3.67	12.54	64	1003	200	14	789		
Aug	17.1	12509	3.78	12.92	26	415	81	6	329		
Sep	17.0	12529	3.79	12.95	22	344	67	5	272	SEER	
COOLING SEASON :					134	2107	415	30	1662	12.75	

SUMMARY

	NWWA	IGSHPA	DIFFERENCE
HEATING ENERGY :	19824	19348	-524 kWh
COOLING ENERGY :	458	445	13 kWh
TOTAL ENERGY :	19282	19793	-512 kWh

TABLE 12 MODEL VALIDATION #3 - ENERGY ANALYSIS COMPARISON
HORIZONTAL SYSTEM - 2 PIPE-PER-TRENCH SIDE-BY-SIDE

NWWA METHOD : PREDICTION WITH INCREASED LOAD* (AND ACTUAL LENGTH)

MONTH	BLDG. LOAD* W	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	7720	13.4	12590	2.35		163	1176	797	73	2047	0	
Nov	8382	6.0	10629	2.12		302	1692	1378	136	3206	0	
Dec	12352	-0.3	8976	1.92		529	2278	2233	238	4841	95	
Jan	13675	-2.0	8531	1.87		644	2555	2652	290	5982	485	
Feb	15661	-2.5	8388	1.85		582	2169	2295	253	5065	348	
Mar	9705	-1.0	8782	1.90		498	2071	2082	224	4394	17	
Apr	7720	2.8	9736	2.01		243	1188	1065	109	2362	0	
May	6396	8.9	11416	2.21		105	655	493	47	1195	0	HSPF
HEATING SEASON :						3045	13781	12996	1370	29091	943	1.90
Jun	7814	17.0	13768	2.71	9.25	23	424	104	10	310		
Jul	7814	27.4	12476	2.24	7.63	50	904	257	23	551		
Aug	6098	24.2	12883	2.39	8.14	38	697	189	17	490		
Sep	6098	18.8	13546	2.63	8.97	13	239	60	6	173	SEER	
COOLING SEASON :						124	2264	610	56	1524	7.82	

IGSHPA METHOD : PREDICTION WITH INCREASED LOAD* (AND ACTUAL LENGTH)

MONTH	BLDG. LOAD* W	EWT PRED'D. C	HP CAP. W	COP	EER	RUN TIME hrs	GROUND EXT./REJ. kWh	HP INPUT kWh	CIRC. PUMP INPUT kWh	BLDG. ENERGY kWh	SUPP. ENERGY kWh	
Oct	7720	10.1	11734	2.25		174	1137	831	78	2047	0	
Nov	8382	4.3	10192	2.07		315	1654	1410	142	3206	0	
Dec	12352	-1.6	8634	1.88		546	2206	2260	246	4841	130	
Jan	13675	-5.1	7693	1.77		676	2259	2635	304	5982	784	
Feb	15661	-5.9	7493	1.74		596	1906	2290	268	5065	601	
Mar	9705	-4.2	7929	1.80		547	1923	2167	246	4394	58	
Apr	7720	0.9	9291	1.98		255	1155	1092	115	2362	0	
May	6396	6.2	10704	2.13		112	633	511	50	1195	0	HSPF
HEATING SEASON :						2618	12872	13197	1448	29091	1574	1.79
Jun	7814	19.6	13448	2.59	8.85	23	429	109	10	310		
Jul	7814	24.2	12874	2.38	8.13	49	886	240	22	551		
Aug	6098	25.0	12778	2.35	8.01	38	700	192	17	490		
Sep	6098	22.7	13083	2.45	8.37	13	245	65	6	173	SEER	
COOLING SEASON :						123	2259	606	55	1524	7.87	

NOTE : THE 'NEW LOAD' IS THE ACTUAL LOAD DIVIDED BY 70%
THE LOAD SHOWN IN 'BLDG LOAD' IS THE MAX. LOAD OCCURING IN THE MONTH

SUMMARY

	NWWA	IGSHPA	DIFFERENCE
HEATING ENERGY :	15309	16219	-910 kWh
COOLING ENERGY :	665	661	4 kWh
TOTAL ENERGY :	15974	16880	-906 kWh

APPENDIX A

Description of the Mathematical Models for Ground Heat Exchanger Length Prediction

FOREWORD

This appendix describes eight models that have been developed to size heat exchangers and for energy analysis of eight ground-source heat pump heat exchanger configurations. The eight configurations are as follows :

- horizontal, single pipe in trench
- horizontal, two pipes in trench (side-by-side)
- horizontal, two pipes in trench (over/under)
- horizontal, four pipes in trench (in square, 2x2)
- vertical, single U-tube
- vertical, two U-tubes (in parallel)
- vertical, three U-tubes (in parallel)
- vertical, four U-tubes (in square - in parallel)

The models permit calculation of he heat pump entering fluid temperature, EWT, for a given configuration at any off-design condition, which allows one to perform an energy analysis. However, the models also size the ground heat exchanger length for given design EWTs. The models can handle two situations.

The first situation implies that the known (or fixed) parameters are the heat exchanger physical characteristics (the total piping length is known), as well as the soil, piping, fluid, design loads and weather characteristics. The unknown (or variable) parameters are the heat pump characteristics, eg. heating and cooling capacities and COPs at design conditions. Two values determining the EWT range, EWT_{min} and EWT_{max}, are set by built-in or rule-of-thumb values in the models, or they can be manual inputs. The heat pump characteristics are then derived from the manufacturer data at the EWT design values and the simulation starts. An iterative solution is required to obtain the EWTs at any off-design condition starting with the manual or computer inputs of EWT_{min} and EWT_{max}. The iteration procedure stops when the convergence criteria are satisfied.

The second situation implies that the known parameters are the heat pump characteristics, at the design conditions. In this situation, EWT_{min} and EWT_{max} are specified. As in the first situation, the soil, piping, fluid, design loads and weather characteristics are fixed. However, the total piping length is the unknown parameter. The models have built-in or rule-of-thumb lengths or allow manual input of the initial length. With this, the simulation can start. An iterative solution is required to obtain optimum total piping length for which the entering fluid temperatures (EWTs) will be predicted at any off-design conditions (between the limits EWT_{min} and EWT_{max}). The iteration procedure stops when the value of the length gives predicted minimum and maximum EWT values within 1 °C of the initial EWT_{min} and EWT_{max}. At that point, the heat exchanger length has been established.

Figures A.1, A.2 and A.3 provide the model flowcharts for the different ground heat exchanger configurations. In each figure, the first situation is **CASE = 1** on the flowchart. The second situation is **CASE = 2**. The iterative solutions correspond to the two loops shown on the far left and right of each figure. **CASE = 1** resets $EWT_{min\,/\,max}$ and **CASE = 2** resets the length L.

Each flowchart identifies the main equation numbers used in the models. Each equation is defined in the following sections of the appendix.

Input Data describes all the input parameters. **Section 2** gives the rule-of-thumb method to determine $EWT_{min\,/\,max}$ and length L initial values, and one method of estimating the heat pump run-time ratio, RTR_t. **Section 3** describes the equations for soil/field resistance, RS_t^I, the length multiplier for pipe-to-pipe interference, $LMLS_t^I$, the surface effect factors, $K1_t^I$ and $K2_t^I$ (applicable to the horizontal systems only), which allow calculation of the difference between the average fluid temperature, Tf, and the far-field temperature, T_∞, at the end of any given period, $(Tf - T_\infty)TOTAL_t$. The average fluid temperature in the heat exchanger at the end of a given period, accounting for seasonal effects and freezing effects (for horizontal systems only), Tf_t, is calculated as described in **Section 4**. Finally, **Section 5** describes the calculation of the entering fluid temperature, EWT_t, at the end of any given period.

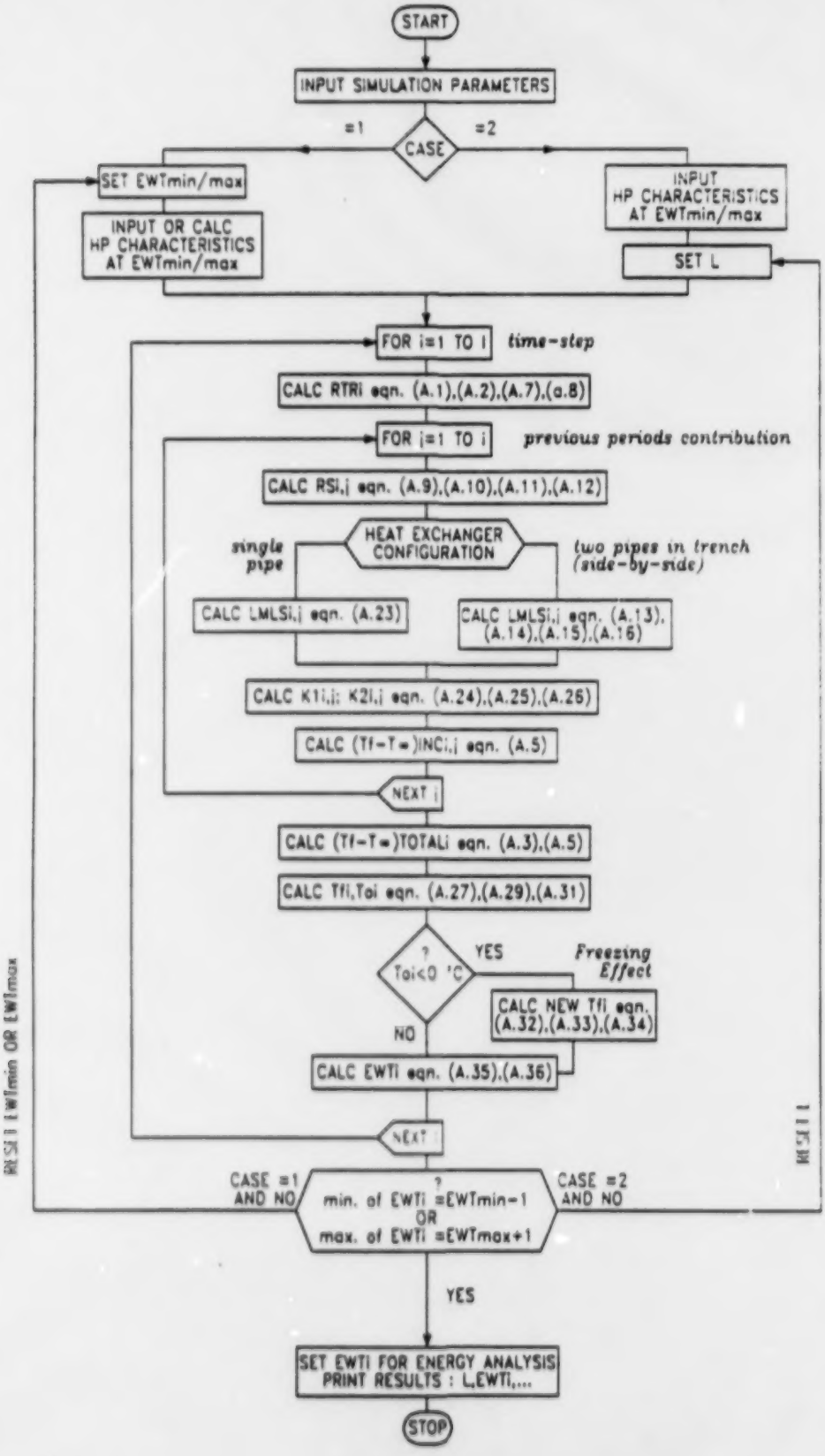


Figure A.1 Flowchart for Horizontal Single Pipe and Two Pipes in Trench Models

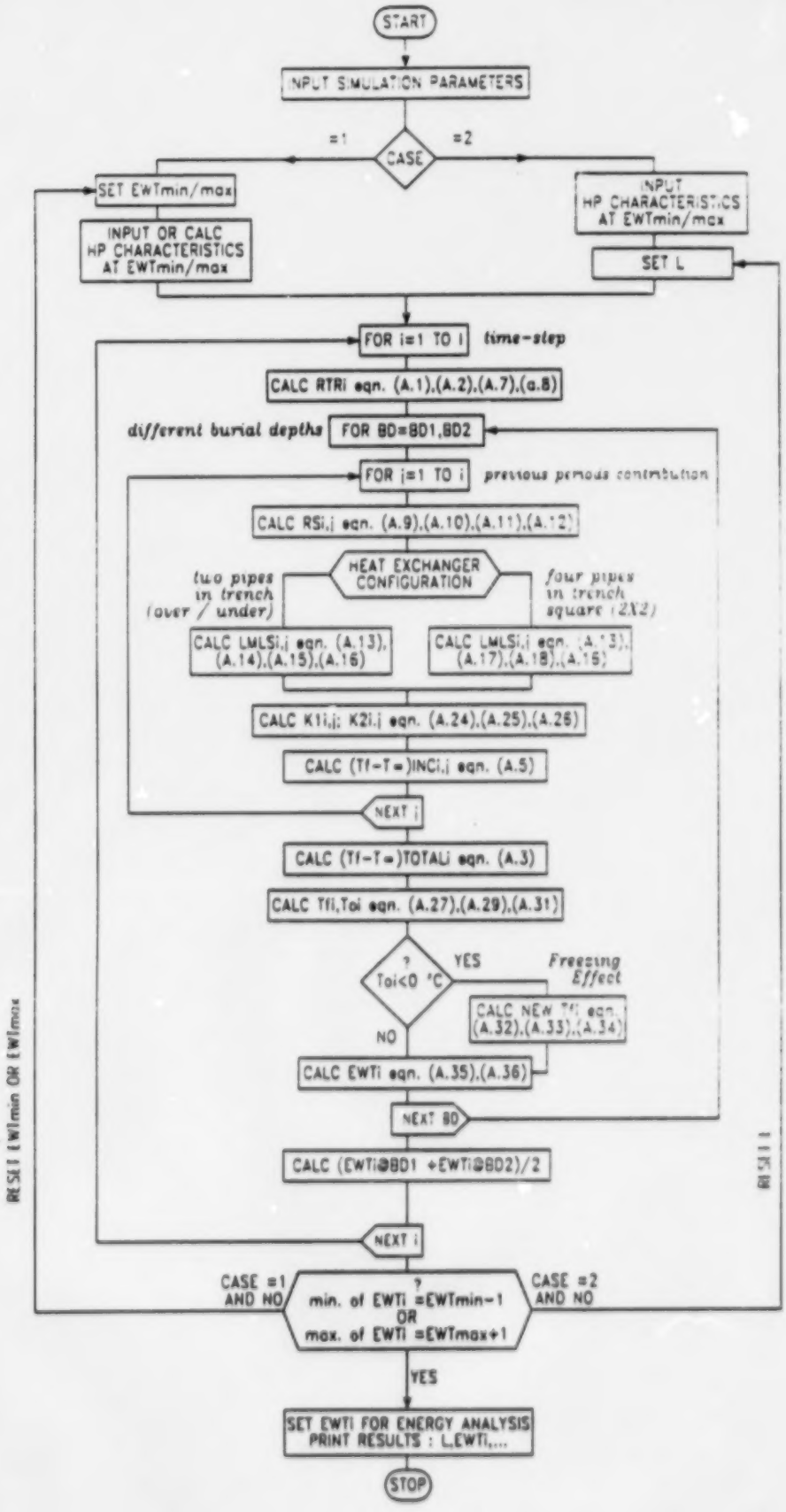


Figure A.2 Flowchart for Horizontal Two Pipes in Trench and Four Pipes in Trench Models

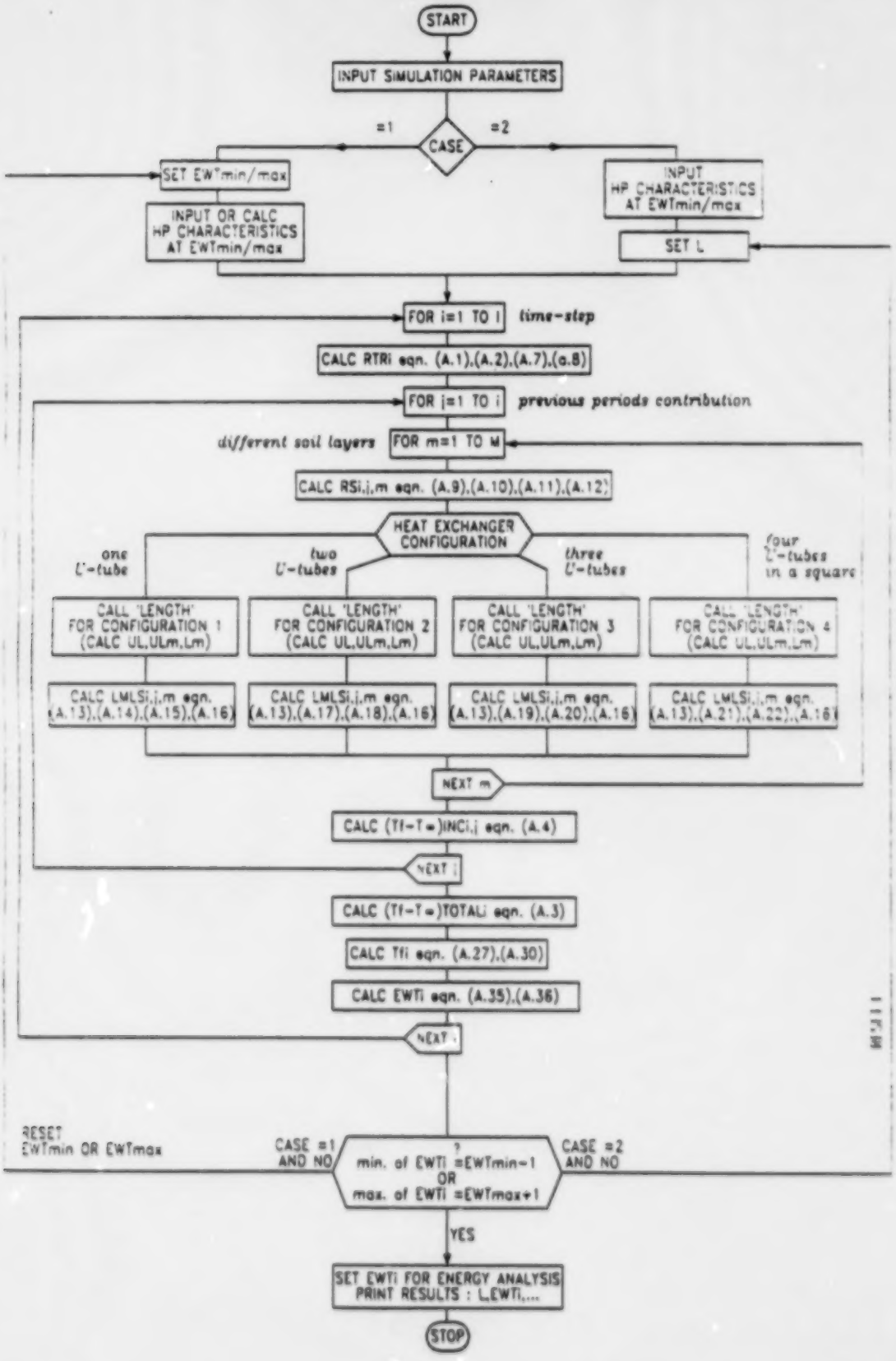


Figure A.3 Flowchart for Vertical One, Two, Three, and Four in a square, U-Tubes Models

1. INPUT DATA

As a preliminary step, one must establish a set of input data, defining all the characteristics of the ground-source heat pump system. These are:

1.1 Soil Characteristics:

T=A, undisturbed mean earth temperature, in °C
SA, annual earth surface temperature amplitude, in °C
t_o, earth surface phase constant (defined as the day since January 1st when minimum earth surface temperature occurred), in days
M, number of different soil layers* (m = 1 to M)
z_m, thickness of each layer (m =1 represents first layer of soil below the earth surface) in m
$k_{i,m}$, soil thermal conductivity at a given period i and at a given layer, m, in W/m°C
$\alpha_{i,m}$, soil thermal diffusivity at a given period i and at a given layer m, in m^2/s
k_{ice}, frozen soil thermal conductivity, in W/m°C

1.2 Heat Exchanger Characteristics:

- horizontal single pipe:
  - BD, burial depth of the pipe, in m
  - L, total piping length, in m
- horizontal, two pipe-per-trench, side-by-side pipes:
  - BD, burial depth of both pipes, in m
  - SD, distance between pipes, in m
  - TL, single pipe length or trench length, in m
  - L, total piping length (= 2 TL), in m
- horizontal, two pipe-per-trench, over/under pipes:
  - BD1, burial depth of the 'top' pipe (closer to the surface), in m
  - BD2, burial depth of the 'bottom' pipe (farther to the surface), in m
  - TL, single pipe length or trench length, in m
  - L, total piping length (= 2 TL), in m
- horizontal, four pipe-per-trench, pipes on corners of a rectangle:
  - BD1, burial depth of the 'top layer' of pipes, in m
  - BD2, burial depth of the 'bottom layer' of pipes, in m
  - SD, distance between pipes at same level, in m
  - TL, single pipe length or trench length, in m
  - L, total piping length (= 4 TL), in m
- vertical, one U-tube:
  - SP, distance between pipes in the U-tube, in m
  - L_o, depth below surface at top of vertical U-tube, in m
  - UL_m, single pipe length in the soil layer m (corresponding to the thickness z_m), in m
  - L_m, total piping length in the soil layer m (= 2 x UL_m), in m
  - UL, total single pipe length (sum of all UL_m), in m
  - L, total piping length (= 2 UL), in m
- vertical, two U-tubes; in one or two boreholes:
  - SP, distance between pipes in U-tubes, in m
  - SU, distance between the U-tubes, in m
  - L_o, depth below surface at top of vertical U-tube, in m
  - UL_m, single pipe length in the soil layer m (corresponding to the thickness z_m), in m

---

*a layer is where the soil or formation characteristics, i.e. thermal conductivity, are uniform.

L_m, total piping length in the soil layer m (= 4 x UL_m), in m
UL, total single pipe length (sum of all UL_m), in m
L, total piping length (= 4 UL), in m

- vertical, three U-tubes, in one, two, or three boreholes:
  SP, distance between pipes in U-tubes, in m
  SU1, distance between U-tubes one and two, in m
  SU2, distance between U-tubes two and three, in m
  SU3, distance between U-tubes three and one, in m
  L_o, depth below surface at top of vertical U-tube, in m
  UL_m, single pipe length in the soil layer m (corresponding to the thickness z_m), in m
  L_m, total piping length in the soil layer m (= 6 x UL_m), in m
  UL, total single pipe length (sum of all UL_m), in m
  L, total piping length (= 6 UL), in m
- vertical, four U-tubes, U-tubes on corners of a rectangle:
  SP, distance between pipes in U-tubes, in m
  SU, short side of the rectangle, in m
  LU, long side of the rectangle, in m
  L_o, depth below surface at top of vertical U-tube, in m
  UL_m, single pipe length in the soil layer m (corresponding to the thickness z_m), in m
  L_m, total piping length in the soil layer m (= 8 x UL_m), in m
  UL, total single pipe length (sum of all UL_m), in m
  L, total piping length (= 8 UL), in m

note: These length inputs are optional, except L_o. The values of UL, and L can be first estimated (see Section 3) and optimized as the introduction explained (check paragraph on iteration procedure in the Foreword).

1.3 Piping and Fluid Characteristics:

ro, outside pipe diameter, in m
ri, inside pipe diameter, in m
k_p, pipe thermal conductivity, in W/m°C
ρ, fluid density, in kg/m^3
C_p, fluid specific heat, in J/kg°C

1.4 Heat Pump Characteristics (optional if EWT's are not known):

EWT_{min}, minimum entering fluid temperature allowed (or suggested) by the manufacturer, in °C
EWT_{max}, maximum entering fluid temperature allowed (or suggested) by the manufacturer, in °C
$CAPH_{min}$, heating mode capacity at the EWT_{min}, in W
$CAPC_{max}$, cooling mode capacity at the EWT_{max}, in W
Q_{HE}, heat extracted at the EWT_{min}, in W
Q_{HR}, heat rejected at the EWT_{max}, in W
m_v, volume flow rate at which heat pump runs, in m^3/s

1.5 Design Load Characteristics:

IDT_H, indoor design temperature in the heating months, in °C
ODT_H, outdoor design temperature in the heating season, in °C
HLOAD, design heating load at ODT_H, in W
IDT_C, indoor design temperature in the cooling months, in °C
ODT_C, outdoor design temperature in the cooling season, in °C
CLOAD, design total cooling load at ODT_C, in W

1.6 Weather Data:

DD_i, degree-days (monthly, weekly or daily) corresponding to a given period i, in °C days to base 18°C

1.7 Simulation Data Characteristics:

i, period increment (day, week or month)
I, simulation period with i increments, i.e. I = 12 months simulation with i = 1 to 12 months

2. FIRST ESTIMATION OF $EWT_{min/max}$, L, RTR_i

In order to start the calculation procedure to determine the total piping length, L, for a given EWT_{min} or $_{max}$, or the $EWT_{min/max}$ corresponding to a given total piping length, L, one must first estimate:

- the minimum and maximum entering fluid temperature, EWT_{min} and EWT_{max};
- the total piping length, L;
- the run time ratio for both heating and cooling periods, i, $HRTR_i$ and $CRTR_i$, respectively.

2.1 Estimation of EWT_{min}, EWT_{max}

If no values have been given by the heat pump manufacturer, and/or the user does not fix limits, then one can use the following values, [1], [2], as a first approximation:

$EWT_{min} = ODT_{hl} + 18$, in °C

note: If the resulting value of EWT_{min} is below the freezing point, then use, EWT_{min} = 0°C, as a first approximation.

EWT_{max} = 35°C

2.2 Estimation of the Total Piping Length, L

An initial value of L needs to be set to start the calculation procedure. Here are suggested values for each heat exchanger configuration, [1], [2]:

- horizontal, single pipe: L = 37 m/kW (130 m/ton)

  note: kW (or ton) is the heat pump cooling capacity. e.g. if heat pump is 10.55 kW (3 ton), then L = 390 m

- horizontal, two pipes-per-trench: L = 44 m/kW (155.5 m/ton)

- horizontal, four pipes-per-trench: L = 54 m/kW (190.5 m/ton)

- vertical, all configurations: L = 35 m/kW (122 m/ton)

2.3 Estimation of the Heat Pump Run-Time Ratio, RTR_i

The run-time ratio, RTR_i, defined as the ratio of the heat pump run-time over the cycle time (or period time i), are first estimated as:

- if the period i is in the winter months, then the heating run-time ratio ($HRTR_i$) is defined as:

(A.1) $$HRTR_i = \frac{DD_i \times HLOAD}{(IDT_H - ODT_H) \times CAPH_{min} \times D_i}$$

where: DD_i, HLOAD, IDT_H, ODT_H, $CAPH_{min}$ have been defined earlier (see **INPUT DATA**)
D_i is the number of days in the period i

- if the period i is in the summer months, then the cooling run-time ratio ($CRTR_i$) is defined as:

(A.2) $$CRTR_i = \frac{DD_i \times CLOAD}{(ODT_C - IDT_C) \times CAPC_{max} \times D_i}$$

where: DD_i, CLOAD, ODT_C, IDT_C, $CAPC_{max}$, D_i have been defined earlier.

3. CALCULATION OF $(Tf - T\infty)TOTAL_i$

The difference between the average fluid temperature, Tf, and the far-field temperature, T∞, at the end of any given period i must now be calculated.

3.1 General Expressions

To obtain the value of the difference between the average fluid temperature, Tf, and the earth far-field temperature, T∞, in the heat exchanger, at the end of any period under consideration, one must add the effects of all time periods up to and including the period under consideration.

Expressed mathematically, the value of $(Tf - T\infty)TOTAL_i$ at the end of the i^{th} period, is the sum of all the incremental contributions from the previous periods j, with j = 1 to j = i (each of the period increments are expressed as $(Tf - T\infty)INC_i^j$).

Thus,

$$(Tf-T\infty)TOTAL_i = (Tf-T\infty)INC_i^1 + (Tf-T\infty)INC_i^2 + \ldots + (Tf-T\infty)INC_i^{j-1} + (Tf-T\infty)INC_i^j$$

or

(A.3) $$(Tf-T\infty)TOTAL_i = \sum_{j=1}^{j=i} (Tf-T\infty)INC_i^j$$

For any given period i, the incremental value, $(Tf - T\infty)INC_i^j$, is given by one of the following expressions:

- **in the case of a vertical system:**

(A.4)
$$(Tf - T_\infty)INC_i^j = \frac{(Q_{HR} \cdot \Delta CRTR_i) - (Q_{HE} \cdot \Delta HRTR_i)}{\sum_{m=1}^{M} \frac{L_m}{(RS_{Lm}^j \cdot LMLS_{(m)}^j) + R_p}}$$

- **in the case of a horizontal system:**

(A.5)
$$(Tf - T_\infty)INC_i^j = (1 - K1_i^j - K2_i^j) \cdot \frac{(Q_{HR} \cdot \Delta CRTR_i) - (Q_{HE} \cdot \Delta HRTR_i)}{\frac{L}{(RS_i^j \cdot LMLS_i^j) + R_p}}$$

3.2 Vertical Systems

The different terms used in the two general expressions above are now explained starting with the terms in the formula (A.4) for the vertical systems:

The terms Q_{HR}, Q_{HE}, L have been described earlier in the **INPUT DATA** section.

The pipe thermal resistance, R_p, is given by the formula:

(A.6)
$$R_p = \frac{Ln\left(\frac{ro}{ri}\right)}{2 \cdot \pi \cdot k_p}$$

where: R_p is expressed in m°C/W;
ro, ri, k_p defined in the **INPUT DATA** section.

The terms $\Delta CRTR_i$ and $\Delta HRTR_i$ are the changes is run-time ratios from one period, i - 1 to another, i, as follows:

(A.7)
$$\Delta CRTR_i = CRTR_i - CRTR_{i-1}$$

(A.8)
$$\Delta HRTR_i = HRTR_i - HRTR_{i-1}$$

The expression for the soil field resistance RS_{Lm}^j at any ending period i, and for any given layer of soil m, is:

(A.9)
$$RS_{Lm}^j = \frac{PWRFOR\left(\frac{ro}{r_{Lm}^j}\right)}{2 \cdot \pi \cdot k_{Lm}}$$

where: the expression of the far-field radius, $r_{\infty}^{j}{}_{,m}$, at the end of the period i for any given layer of soil, m, is the following:

(A.10)
$$r_{\infty}^{j}{}_{,m} = 4\sqrt{\alpha_{i,m} \cdot t_i^j}$$

where: t_i^j being the time, expressed in seconds, corresponding from period j to period i. Expressed mathematically, t_i^j is:

(A.11)
$$t_i^j = (I - j + 1) \times \textit{conversion factor (seconds)}$$

example: I = 12 weeks, simulation period, i being a weekly period. If one calculates the contribution of the 5th period (j = 5) to the calculation of the average fluid temperature at the end of the 10th period (i = 10), then t_i^j must be calculated as $t_{10}^5 = (10 - 5 + 1) \times 7 \times 24 \times 3600$ which is 3628800 seconds.

The function PWRFOR(X) is the line source integral expressed as a power series, as follows:

(A.12)
$$PWRFOR(X) = Ln\left(\frac{1}{X}\right) - 0.981755 \cdot \sum_{n=1}^{N} \frac{(-1)^{n-1}}{2n \cdot n!}(2 \cdot X)^{2n}$$

This function will be used later for different variables X. In the case of the soil/field resistance, the variable X is equal to ro divided by $r_{\infty}^{j}{}_{,m}$ as indicated in the formula (A.9). It can be demonstrated that 13 terms (n = 13) are enough to provide good accuracy, for the formulas used in this appendix.

The factor for pipe-to-pipe interference, $LMLS_{i,m}^{j}$, is the product of two multipliers, $LMLSHE_{i,m}^{j}$, the multiplier for heat exchange equivalency, and $LMLST_{\infty}^{j}{}_{,m}$, the multiplier for far-field temperature equivalency:

(A.13)
$$LMLS_{i,m}^{j} = LMLSHE_{i,m}^{j} \cdot LMLST_{\infty}^{j}{}_{,m}$$

and

(A.13a)
$$\textit{if } LMLSHE_{i,m}^{j} \cdot LMLST_{\infty}^{j}{}_{,m} > 2 \quad \textit{then} \quad LMLS_{i,m}^{j} = 2$$
$$\textit{if } LMLSHE_{i,m}^{j} \cdot LMLST_{\infty}^{j}{}_{,m} < 1 \quad \textit{then} \quad LMLS_{i,m}^{j} = 1$$

The expressions of both multipliers are given below for each vertical configuration considered in this appendix.

3.2.1 One U-tube in a borehole:

The multipliers are given by the expressions:

$$\text{Define: } X = \frac{ro}{r_{eq}^{l}} \text{ and } Y = \frac{SP}{r_{eq}^{l}}$$

(A.14)

$$\text{if } Y \geq 1 \text{ then } LMLSHE_{eq}^{l} = 1$$
$$\text{if } Y < 1 \text{ then } LMLSHE_{eq}^{l} = 1 - \frac{PWRFOR(Y)}{PWRFOR(X)}$$

(A.15)

$$\text{if } \frac{Y}{2} \geq 1 \text{ then } LMLST_{eq}^{l} = 1$$
$$\text{if } \frac{Y}{2} < 1 \text{ then } LMLST_{eq}^{l} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right)}$$

where the function SINACOS() is defined as :

(A.16)

$$SINACOS\left(\frac{Y}{2}\right) = \frac{\cos^{-1}\left(\frac{Y}{2}\right)}{\pi} - \left(\frac{Y}{2}\right) \cdot \frac{\sin\left(\cos^{-1}\left(\frac{Y}{2}\right)\right)}{\pi}$$

with cosine and sine functions expressed in radians

3.2.2 Two U-tubes in the same borehole or two different boreholes

The multipliers are given by the expressions:

$$\text{Define: } X = ro / r_{eq}^{l}$$
$$Y = SP / r_{eq}^{l}$$
$$Z = SU / r_{eq}^{l}$$
$$W = SH / r_{eq}^{l}$$
$$\text{where } SH = \sqrt{SP^2 + SU^2}, \text{ distance between two opposite pipes}$$

(A.17)

$$\text{if } Y \geq 1 \text{ then } LMLSHE_{eq}^{l} = 1$$
$$\text{if } Z \geq 1 \text{ then } LMLSHE_{eq}^{l} = 1 - \frac{PWRFOR(Y)}{PWRFOR(X)}$$
$$\text{if } W \geq 1 \text{ then } LMLSHE_{eq}^{l} = 1 - \frac{PWRFOR(Y)}{PWRFOR(X)} - \frac{PWRFOR(Z)}{PWRFOR(X)}$$
$$\text{if } W < 1 \text{ then } LMLSHE_{eq}^{l} = 1 - \frac{PWRFOR(Y)}{PWRFOR(X)} - \frac{PWRFOR(Z)}{PWRFOR(X)} - \frac{PWRFOR(W)}{PWRFOR(X)}$$

and

$$
\begin{aligned}
&\text{if } \frac{Y}{2} \geq 1 \text{ then } LMLST = l_{cm} = 1\\
&\text{if } \frac{Z}{2} \geq 1 \text{ then } LMLST = l_{cm} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right)}\\
&\text{if } \frac{W}{2} \geq 1 \text{ then } LMLST = l_{cm} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(0.5 \cdot SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right)}\\
&\text{if } \frac{W}{2} < 1 \text{ then } LMLST = l_{cm} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(0.5 \cdot SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right) - \left(0.5 \cdot SINACOS\left(\frac{W}{2}\right) \cdot \frac{PWRFOR\left(\frac{W}{2}\right)}{PWRFOR(X)}\right)}
\end{aligned}
\qquad \text{(A.18)}
$$

where the function SINACOS( ) has been previously defined, see expression (A.16)

3.2.3 Three U-tubes in the same borehole or any combination of three boreholes

note: as indicated in the **INPUT DATA** section, the three distances between the U-tubes are given by the user (SU1, SU2, SU3). For the purpose of the length multipliers calculation, only the lowest two values of the three distances are used. Let's call the two lowest values SD1, SD2.

$$
\begin{aligned}
\text{Define: } X &= rb / l_{cm}\\
Y &= SP1 / l_{cm}\\
Z &= SD11 / l_{cm}\\
W &= SD21 / l_{cm}
\end{aligned}
$$

and the multipliers expressions are:

$$
\begin{aligned}
&\text{if } Y \geq 1 \text{ then } LMLSHE_{cm}^{l} = 1\\
&\text{if } Z \geq 1 \text{ then } LMLSHE_{cm}^{l} = 1 + \frac{PWRFOR(Y)}{PWRFOR(X)}\\
&\text{if } W \geq 1 \text{ then } LMLSHE_{cm}^{l} = 1 + \frac{PWRFOR(Y)}{PWRFOR(X)} + \frac{2 \cdot PWRFOR(Z)}{PWRFOR(X)}\\
&\text{if } W < 1 \text{ then } LMLSHE_{cm}^{l} = 1 + \frac{PWRFOR(Y)}{PWRFOR(X)} + \frac{2 \cdot PWRFOR(Z)}{PWRFOR(X)} + \frac{2 \cdot PWRFOR(W)}{PWRFOR(X)}
\end{aligned}
\qquad \text{(A.19)}
$$

(A.20)

$$\text{if } \frac{Y}{2} \geq 1 \text{ then } LMLST=l_{i,m} = 1$$

$$\text{if } \frac{Z}{2} \geq 1 \text{ then } LMLST=l_{i,m} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right)}$$

$$\text{if } \frac{W}{2} \geq 1 \text{ then } LMLST=l_{i,m} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right)}$$

$$\text{if } \frac{W}{2} < 1 \text{ then } LMLST=l_{i,m} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right) - \frac{1}{\left(SINACOS\left(\frac{W}{2}\right) \cdot \frac{PWRFOR\left(\frac{W}{2}\right)}{PWRFOR(X)}\right)}}$$

for the special case: if SD1 = SD2 (= SD) formulas (A.19) and (A.20) become respectively (A.19a) and (A.20a) :

(A.19a)

$$\text{if } Y \geq 1 \text{ then } LMLSHE^{l}_{i,m} = 1$$

$$\text{if } Z \text{ or } W \geq 1 \text{ then } LMLSHE^{l}_{i,m} = 1 + \frac{PWRFOR(Y)}{PWRFOR(X)}$$

$$\text{if } Z \text{ or } W < 1 \text{ then } LMLSHE^{l}_{i,m} = 1 + \frac{PWRFOR(Y)}{PWRFOR(X)} + \frac{4 \cdot PWRFOR(Z)}{PWRFOR(X)}$$

(A.20a)

$$\text{if } \frac{Y}{2} \geq 1 \text{ then } LMLST=l_{i,m} = 1$$

$$\text{if } \frac{Z}{2} \text{ or } \frac{W}{2} \geq 1 \text{ then } LMLST=l_{i,m} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right)}$$

$$\text{if } \frac{Z}{2} \text{ or } \frac{W}{2} < 1 \text{ then } LMLST=l_{i,m} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(2 \cdot SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right)}$$

see expression (A.16) for the function SINACOS( ).

3.2.4 Four U-tubes in the same borehole or in two or four different boreholes (in all cases U-tubes are on the corners of a rectangle)

Define: $X = ro|_{r=l_{i,m}}$

$Y = SP|_{r=l_{i,m}}$

$Z = SU|_{r=l_{i,m}}$

$W = LU|_{r=l_{i,m}}$

$V = SH|_{r=l_{i,m}}$

where $SH = \sqrt{SU^2 + LU^2}$, *distance between two opposite bores*

Then, the multipliers have the following expressions:

(A.21)

$$\begin{aligned}
&\text{if } Y \geq 1 \text{ then } LMLSHE_{lm} = 1\\
&\text{if } Z \geq 1 \text{ then } LMLSHE_{lm} = 1 \cdot \frac{PWRFOR(Y)}{PWRFOR(X)}\\
&\text{if } W \geq 1 \text{ then } LMLSHE_{lm} = 1 \cdot \frac{PWRFOR(Y)}{PWRFOR(X)} \cdot \frac{2 \cdot PWRFOR(Z)}{PWRFOR(X)}\\
&\text{if } V \geq 1 \text{ then } LMLSHE_{lm} = 1 \cdot \frac{PWRFOR(Y)}{PWRFOR(X)} \cdot \frac{2 \cdot PWRFOR(Z)}{PWRFOR(X)} \cdot \frac{2 \cdot PWRFOR(W)}{PWRFOR(X)}\\
&\text{if } V < 1 \text{ then } LMLSHE_{lm} = 1 \cdot \frac{PWRFOR(Y)}{PWRFOR(X)} \cdot \frac{2 \cdot PWRFOR(Z)}{PWRFOR(X)} \cdot \frac{2 \cdot PWRFOR(W)}{PWRFOR(X)} \cdot \frac{2 \cdot PWRFOR(V)}{PWRFOR(X)}
\end{aligned}$$

and

(A.22)

$$\begin{aligned}
&\text{if } \frac{Y}{2} \geq 1 \text{ then } LMLST_{lm} = 1\\
&\text{if } \frac{Z}{2} \geq 1 \text{ then } LMLST_{lm} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right)}\\
&\text{if } \frac{W}{2} \geq 1 \text{ then } LMLST_{lm} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right)}\\
&\text{if } \frac{V}{2} \geq 1 \text{ then } LMLST_{lm} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{W}{2}\right) \cdot \frac{PWRFOR\left(\frac{W}{2}\right)}{PWRFOR(X)}\right)}\\
&\text{if } \frac{V}{2} < 1 \text{ then } LMLST_{lm} = \frac{1}{1 - \left(0.5 \cdot SINACOS\left(\frac{Y}{2}\right) \cdot \frac{PWRFOR\left(\frac{Y}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{Z}{2}\right) \cdot \frac{PWRFOR\left(\frac{Z}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{W}{2}\right) \cdot \frac{PWRFOR\left(\frac{W}{2}\right)}{PWRFOR(X)}\right) - \left(SINACOS\left(\frac{V}{2}\right) \cdot \frac{PWRFOR\left(\frac{V}{2}\right)}{PWRFOR(X)}\right)}
\end{aligned}$$

again, the expression for the function SINACOS( ) is given by (A.16)

3.3 Horizontal Systems

For the horizontal systems, the terms in the formula (A.5) are described in the following.

The terms Q_{HR}, Q_{HE}, and L are defined and values are given in the first section, **INPUT DATA**.

The expressions for the pipe thermal resistance, R_o, and the cooling and heating changes in run-time-ratios, ΔCRTR and ΔHRTR, are given by the formulas (A.6), (A.7) and (A.8) respectively.

The expression of the soil/field resistance RS_i^l is equivalent to the formula (A.9) for the vertical systems. The difference is that the index m (subscript) from (A.9) is dropped because the horizontal heat exchanger remains in the same soil layer along its entire length. This implies also that soil characteristics $k_{s,m}$ and $a_{s,m}$ (thermal conductivity and diffusivity) become k_s and a_s, characteristics of the top soil layer, and the soil far-field radius will become r_{ss}.

Also, the expression for the pipe-to-pipe interference, $LMLS_i^l$ (notice that the index m has been dropped) is equivalent to the expression (A.13). Therefore, the expressions of the multipliers for the horizontal configurations are similar to those in the vertical sections, as it follows:

3.3.1 One pipe in a trench:

There is no interference, therefore:

(A.23)

$$LMLS_i^l = 1$$

3.3.2 Two pipes-per-trench or two trenches/one pipe-per-trench

These configurations correspond to the cases described in the **INPUT DATA** section as "side-by-side" and "over/under". In both cases, expressions developed in the subsection for one U-tube in a borehole can be used. The formulas (A.13), (A.14), (A.15) and (A.16) will be applied with SP = BD2 - BD1, in the case of two pipes over/under.

3.3.3 Four pipes-per-trench, pipes on corners of a rectangle

The expressions (A.13), (A.17), (A.18) and (A.16) used in the subsection "two U-tubes in the same borehole or two different boreholes", can be applied with the following change in variables:

SP becomes SD as defined in INPUT DATA section
SU becomes BD2 - BD1 as defined in INPUT DATA section
$SH = \sqrt{(BD2 - BD1)^2 + SD^2}$

The expressions for the earth's surface proximity effects are developed only in the case of horizontal heat exchangers. The expression $(1 - K1_i^l - K2_i^l)$ in the formula (A.5) represents the correction factor that applies to semi-infinite heat exchange medium, $K1_i^l$ being the factor for the surface effect on the pipe, and $K2_i^l$ for the surface effect on the average far-field temperature at the burial depth. Expressed mathematically, these factors are:

Define $X = ro/r_{ss}^l$ and $Y = BD/r_{ss}^l$

- $K1_i^l$, factor for surface effect on a pipe buried at the depth BD

(A.24)

$$\text{If } 2Y \geq 1 \text{ then } K1_i^l = 0$$
$$\text{If } 2Y < 1 \text{ then } K1_i^l = \frac{PWRFOR(2Y)}{PWRFOR(X)}$$

- $K2_i^l$, factor for surface effect on average far-field temperature at BD

(A.25)

$$\text{If } Y \geq 1 \text{ then } K2_i^l = 0$$
$$\text{If } Y < 1 \text{ then } K2_i^l = 0.5 \cdot COSACOS(Y) \cdot \frac{PWRFOR(Y)}{PWRFOR(X)}$$

where the function COSACOS( ) is defined as:

(A.26) $$COSACOS(Y) = \frac{\cos^{-1}(Y)}{\pi} - Y \cdot \frac{\sin \cdot (\cos^{-1}(Y))}{\pi}$$

with cosine and sine functions expressed in radians

4. CALCULATION OF Tf_i

Average fluid temperature in the heat exchanger at the end of the i^{th} period, with seasonal effects and freezing effect (for horizontal only), is calculated as described in this section.

4.1 General Expression

To obtain the value of the average fluid temperature in the heat exchanger (both horizontal and vertical) at the end of any period under consideration, Tf_i, one would add the three following components: the difference between the average fluid temperature and the far-field temperature; $(Tf - T_\infty)TOTAL_i$; the seasonal earth temperature change, ΔT_∞, and the average far-field temperature, $T_\infty A$. That is:

(A.27) $$Tf_i = (Tf - T_\infty)TOTAL_i + \Delta T_{\infty i} + T_\infty A$$

where $(Tf - T_\infty)TOTAL_i$, (see (A.4) and (A.5) in section 3);
$T_\infty A$, see section 1, **INPUT DATA**;
$\Delta T_{\infty i}$, the seasonal earth temperature change at any given period i of the year.

The expressions of $\Delta T_{\infty i}$ for both horizontal and vertical, are derived from the formula determined by Kusuda, [3], Van Wijk, [4] and Labs, [5], work, which give the temperature on any given day t, at any given depth X, using the following expression:

(A.28) $$T(X,t) = T_\infty A - SA \cdot \exp^{aX} \cdot \cos(b + aX)$$

$$\text{where } a = \sqrt{\frac{\pi}{365\alpha}} \text{ and } b = \frac{2\pi}{365}(t - t_o)$$

a,b and cosine expressed in radians

and where $T_\infty A$, SA, t_o as defined in **INPUT DATA**;
t is calendar time in days with t = 0 at midnight, January 1st;
α the most representative α_i for the full year (or the mean).

The expressions for the seasonal earth temperature changes at any given period i are:

for the horizontal systems:

(A.29) $$\Delta T_{\infty i} = -SA \cdot \exp^{+BD} \cdot \cos(b + aBD)$$

where: SA, t, t_o, a, b, cosine and e have been explained previously;
BD, burial depth of the horizontal pipe.

for the vertical systems:

An approximate expression of ΔT_{∞} is given for the vertical systems. It is the average deviation from $T_{\infty}A$ over depths from L_o, top of the heat exchanger, to L_b, bottom of the heat exchanger, at any particular period i:

(A.30)
$$\Delta T_{\infty_i} = \frac{SA \cdot \exp^{aL_o}}{2 \cdot a \cdot (L_b - L_o)}(\cos(b + aL_o) + \sin(b + aL_o))$$

where: SA, t, t_o, a, b, cosine and e same as before;
sine function expressed in radians;
L_o, depth below surface of top of heat exchanger;
L_b, depth below surface of bottom of heat exchanger ($L_b = L_o + UL$).

4.2 Freezing Effect Approximation

For horizontal systems only, formulas are presented to take into account the freezing effect on the soil surrounding the pipe. The procedure involves a check on the pipe outside temperature at any given period i, To, to determine if it is below the freezing point. If yes, then an estimation of the diameter of the ice and earth ring surrounding the pipe is undertaken to calculate the new average fluid temperature at any given ending period i.

The outside pipe temperature at the end of any given period i is obtained from the following formula:

(A31)
$$To_i = Tf_i - \frac{(Q_{HR} \cdot CRTR)_i - (Q_{HE} \cdot HRTR)_i}{\frac{L}{R_p}}$$

where: all the terms have been defined in the previous sections.

note: (A31) implies that $(To - T_{\infty})TOTAL_i = To_i - (\Delta T_{\infty_i} + T_{\infty}A)$

If To_i, formula (A.31), is negative, then one must estimate ice and earth ring radius, r ice, as follows:

(A.32)
$$r\ ice_i = \frac{5.34 \cdot BD}{\exp\left(\left(\frac{-(\Delta T_{\infty_i} + T_{\infty}A)}{(To - T_{\infty})TOTAL_i} \cdot PWRFOR(X_{ice})\right) + 0.981755\right)}$$

where: $X_{ice} = ro/(5.34\ BD)$;
PWRFOR( ) function see (A.12);
(all other terms and expressions previously described).

If To_i is negative, one must recalculate the average fluid temperature in the heat exchanger at the end of any given period i, Tf_i, using the following expression:

(A33)
$$Tf_i = \frac{(Q_{HR} \cdot CRTR_i) - (Q_{HE} \cdot HRTR_i)}{\frac{L}{RS_{ice,i}} + R_p}$$

where: Q_{HR}, Q_{HE}, $CRTR_i$, $HRTR_i$, L and R_p defined in earlier sections

(A.34)
$$RS_{ice,i} = \frac{PWRFOR(Y_{ice})}{2 \cdot \pi \cdot k_{ice}}$$

where: $Y_{ice} = ro/r\ ice_i$
and k_{ice} is the thermal conductivity of the ice and earth ring (see INPUT DATA).

5. CALCULATION OF EWT_i

This section describes the calculation of the entering fluid temperature at any given period i, EWT_i (heat pump inlet assumed to be equivalent to heat exchanger outlet).

5.1 General expression

The general expression is given by the following formula

(A.35)
$$EWT_i = Tf_i + 0.5 \cdot \frac{(Q_{HR} \cdot CRTR_i) - (Q_{HE} \cdot HRTR_i)}{m \cdot C_p}$$

where: m is the fluid mass flow rate, obtained from the following expression:

(A36)
$$m = m_v \cdot \rho$$

where: both m_v, volume flow rate and ρ, fluid density are given (see INPUT DATA)
and C_p, is the fluid specific heat (see INPUT DATA).

5.2 Horizontal Systems

In the case of a horizontal system where the pipes are buried at two different depths, the EWT_i calculation must be done for both depths, as shown in the figure A.2. The flowchart of this figure indicates that a first EWT_i calculation is done at the burial depth BD1, and then redone with the second burial depth BD2 (equations for factors K1, K2 are dependent of the depth). Finally the entering fluid temperature EWT_i for the system is calculated as the average value of both EWT_i at BD1 and EWT_i at BD2.

6. REFERENCES

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APPENDIX B

Description of the Mathematical Models for Lake-Loop Heat Exchanger Length Prediction

FOREWORD

This appendix describes a simplified model developed to size heat exchangers and for energy analysis of lake-loop heat pump system. The model has three components; the heat-pump, the plastic heat exchanger and the lake. The fluid in the coil permits energy transfer from the heat pump to the lakewater (cooling mode) or from the lakewater to the heat pump (heating mode). The model presented here assumes a lake with the following characteristics:

- Shallow (< 10m), medium-size, **unstratified**;
- Slow temperature changes during a given period which allow one to consider a constant average temperature during that period.

Section 1, **INPUT DATA**, will describe all necessary parameters used in the formulae. The calculation of the lake temperature, LT, and the entering fluid temperature, EWT, to the heat pump for a given period, will be described in **Section 2** and **Section 3**, respectively. **Section 4** will describe the procedure to size the heat exchanger and to perform the energy analysis.

1. INPUT DATA

Here are presented the input parameters necessary to describe the lake, coil and heat pump characteristics.

1.1 Lake Characteristics

Z_l, average (or arbitrary) lake depth, ft

Ap lake surface area, ft^2

S_l, length of lake perimeter, ft

U_l, lake fetch, ft

ρ_{lw}, lakewater density, lb_m/ft^3

Cp_{lw}, lakewater specific heat, $Btu/lb_m \cdot °F$

ν_{lw}, lakewater kinematic viscosity, ft^2/s

Z_g, depth to water table, ft

k_g, ground (surrounding lake) thermal conductivity, $Btu/h \cdot °F \cdot ft^2$

1.2 Lake-Loop Heat Exchanger and Fluid Characteristics

OD, outside pipe diameter, ft

ID, inside pipe diameter, ft

L, total piping length, ft

ρ_f, fluid density, lb_m/ft^3

ν_f, fluid dynamic viscosity, $(lb_f \cdot s)/ ft^2$ (or Poise)

k_f, fluid thermal conductivity, $Btu/h \cdot °F \cdot ft$

Cp_f, fluid specific heat, $Btu/lb_m \cdot °F$

V_f, fluid velocity, ft/s

1.3 Weather Data

Is, intensity of direct solar radiation on a horizontal surface, $Btu/h \cdot ft^2$

e_s, partial pressure of water vapor at lake surface, mmHg

e_a, partial pressure of water vapor in air, mmHg

W_{so}, free stream wind velocity, mph

T_{DB}, dry-bulb air temperature, °F (or °R)

β_a, atmospheric radiation factor, function of cloud cover and relative humidity

σ, Stefan-Boltzman constant, equal to $0.1714 \cdot 10^{-8}$ Btu/h· ft².°R⁴

1.4 Heat Pump Characteristics

Q_{rejE}, heat extracted from the lake at EWT, Btu/h

Q_{rejS}, heat rejected to the lake at EWT, Btu/h

2. LAKE TEMPERATURE CALCULATION

A simplified approach for unstratified lake [1] allows one to calculate the lakewater temperature at the end of a period, LT_f, as a function of the lake temperature at the beginning of the same period, LT_i, and of all the incoming or leaving heat fluxes. These heat fluxes are, Is, solar insolation, Q_R, rate of heat extraction or rejection (heat pump) and Q_{evap}, Q_{conv}, Q_{rad}, Q_G, the evaporative, convective, radiative and ground heat flows, respectively. The following expression gives then the bulk lake temperature changes for a given period.

(B.1) $$LT_f = LT_i + \{(Is + Q_R - Q_{evap} - Q_{conv} - Q_{rad} - Q_G)/m_{lw}\, Cp_{lw}\}$$

(LT expressed in degree Fahrenheit, °F)

where the different terms are defined as:

(B.2) $Q_R = + Q_{rejS}$, if cooling mode

$Q_R = - Q_{rejE}$, if heating mode

(B.3) $$Q_{evap} = (25.1 \cdot e^{0.333}) + \{(690 \cdot W\infty \cdot e)/ Re^{0.2}\}$$

where

$e = e_s - e_a$

$Re = W\infty \cdot LJ / \nu_{lw}$, Reynolds number.

(B.4) $$Q_{conv} = (1.004 + 0.3 \cdot W\infty) \cdot (T_{DB} - LT_i)$$

(B.5) $$Q_{rad} = 0.97 \cdot \sigma \cdot (LT_i^4 - \beta_a\, T_{DB}^4)$$

Note: for the formula (B.5) only, the temperatures LT_i and T_{DB} are in degree Rankine, °R

(B.6) $$Q_G = \{(0.99 \cdot k_g/Zg) + (0.90 \cdot k_g \cdot S/Ap)\} \cdot (LT_i - Tg)$$

Note: Tg is supposed to be an "average" ground temperature.

(B.7) $$m_{lw} = \rho_{lw} \cdot Z \cdot Ap$$

3. ENTERING FLUID TEMPERATURE CALCULATION

With suitable assumptions it is possible to derive an expression from the log mean temperature difference method (LMTD method) for the present heat exchanger configuration. The assumptions are as follows:

- the lakewater is still around the pipe.

- the lakewater temperature is assumed constant during the period considered. Its value, LT, will be equal to:

(B.8) $$LT = (LT_i + LT_f)/2$$

- the overall heat transfer coefficient inside the pipe, h, the fluid mass flow rate, MFR, and specific heat Cp_f, are assumed to remain constant over the entire pipe length.

- there is no heat loss or gain external to the coil and there is no axial conduction along the heat exchanger.

The entering fluid temperature EWT (entering the heat pump is same as leaving the heat exchanger) can be shown to be a function of the leaving fluid temperature, LWT, the average bulk lake temperature, T, and the heat exchanger characteristics, h, L as follows:

(B.9) $$EWT = LT + \{(LWT - LT) \cdot \exp(-h_i \cdot A/MFR \cdot Cp_f)\}$$

(EWT expressed in degree Fahrenheit, °F)

where the different terms are expressed as:

(B.10) $$h_i = Nu \cdot k_f/ID$$

The formula for Nusselt number is given for turbulent flow only ($Re_D > 10^4$):

(B.11) $$Nu = 0.023 \cdot Re_D^{0.8} \cdot Pr^{0.333}$$

The Reynolds, Re_D, and Prandtl, Pr, numbers are given by the formulas:

(B.12) $$Re_D = \rho_f \cdot V_f \cdot ID/\mu_f$$

(B.13) $$Pr = \mu_f \cdot Cp_f / k_f$$

The heat exchanger surface is expressed as A:

(B.14) $$A = 2 \cdot \pi \cdot ID \cdot L$$

and the mass flow rate, MFR, is calculated as follows:

(B.15) $$MFR = \pi \cdot ID^2 \cdot Vf/4$$

4. CALCULATION PROCEDURE

At the beginning of the simulation period, one can assume that both lakewater and fluid temperatures are the same and would be equal to the lake undisturbed temperature. The initial values are then expressed as:

(B.16) $$\text{at } t = o \qquad EWT = LWT = LT_i = LT_f = LT_o$$

As with ground heat exchanger sizing, the model user can specify a minimum EWT to ensure that the system COP does not drop below a certain value. By varying the specified heat exchanger length L, the user can change the values of EWT for each simulation step.

For the first time period, B.1 will yield the lake temperature; B.8 and B.9 will yield the entering liquid temperature to the heat pump. For the second time period, the lake temperature is assumed equal to the

value at the end of the first period and the leaving liquid temperature is equal to the entering liquid temperature at the end of the first period plus the temperature difference across the heat pump heat exchanger, ΔIVT.

At the end of the second time period, B.1 will once again yield the lake temperature; B.8 and B.9 the entering liquid temperature to the heat pump. This process of calculation is repeated for the remaining time steps. If the value of EWT assumes a lower value than desired, a new length L can be selected and the calculation repeated.

5. REFERENCE

Pezent, M.C. and S.P. Kavanaugh. 1990. "Development and verification of a thermal model of lakes used with water source heat pumps." ASHRAE transactions, Vol. 96, Pt. 1, pp. 574-582.